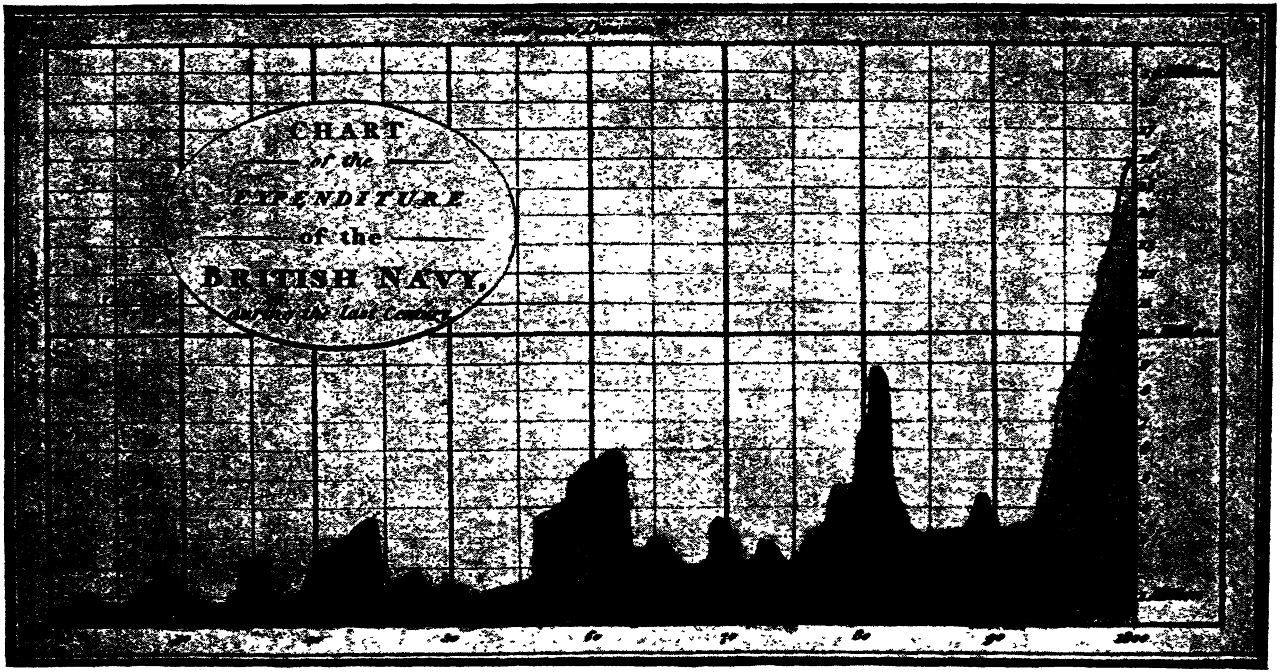

GRAPHS

HOW TO MAKE AND USE THEM



Reproduced from Playfair, W., "The Commercial and Political Atlas Representing by Means of Stained Copper-Plate Charts, The Progress of the Commerce, Revenues, Expenditures, and Debts of England during the Whole of the Eighteenth Century," Third Edition, 1801

AN EARLY GRAPH (1801)

GRAPHS

HOW TO MAKE AND USE THEM

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Revised Edition

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GRAPHS: HOW TO MAKE AND USE THEM

E-Y

CONTENTS

LIST OF ILLUSTRATIONS

PREFACE

Little attempt has been made to standardize practice in the construction of graphs since the *Report of the Joint Committee on Standards for Graphic Presentation*, of the American Statistical Association, in 1915. The report of the committee largely dealt with the standardization of the principles involved rather than the form of the graph or the details of construction.

Consequently, authors dealing with the subject have differed sharply. The present volume points out what the authors believe to be the best current technique, based on a wide survey of representative experience. This will perhaps excuse any apparent dogmatism as to the rules laid down for good procedure.

Herbert Arkin
Raymond R. Colton

College of the City of New York,
July, 1935

GRAPHS

HOW TO MAKE AND USE THEM

CHAPTER I

THE PRINCIPLE OF THE GRAPH

History of the Graph—Purpose of the Graph—Principle of the Graph—The Axes—The Quadrants—Plotting a Point.

Ever since records were first kept, man has been confronted with the problem of the interpretation of numerical data. The preparation of tables was a great step towards facilitating the analysis of figures. However, even the most carefully prepared table of statistics proved inadequate as a satisfactory means by which the significance of data could be quickly, clearly, and easily understood. The graph, which converts into visual form a comparison of various quantities, met just such a need. William Playfair, the first to graphically portray economic data, in 1786 said, "that I have succeeded in proposing a new and useful mode of stating accounts has been so generally recognized. . . . As much information may be obtained in five minutes as would require whole days to imprint on the memory, in a lasting manner, by a table of figures."

History of the Graph. In 1637 René Descartes described for the first time the principle on which the modern graph is based. Through the use of lines drawn perpendicular to one another he was able to represent the values of pairs of numbers by the use of points on the resulting plane.

William Playfair, an English statistician, who has been called the inventor of graphic statistics, published a volume called *A Commercial and Political Atlas* (London 1786, 1787, and 1801),

in which was "Represented by Means of Stained Copper-Plate Charts, the Progress of the Commerce, Revenues, Expenditures, and Debts of England during the Whole of the Eighteenth Century." In his third edition, published in 1801, Playfair refers to the extensive use of charts in the studies of history, genealogy, and chronology. He claimed only to have been the first who applied the principles of geometry to matters of finance, and "it had long before been applied to chronology with great success." Previous to the publication of Playfair's work, Chrome of Giessen, Germany, had graphed, in 1784, the sizes of the European states. At about this time a number of others, as Beaufort, in France in 1789; Gaspari and Boetticher, in Germany in 1789; and von Hoeck in 1794, employed graphs in presenting statistical data.

Lalanna, the French engineer, brought out the principles of nomography in 1842.

Minard, a French engineer, "explained and defended the graph" in a memoir in 1861.

D'Ocagne, called the author of nomography, first published parts of his work in 1891.

John B. Peddle wrote in 1910 the first volume on graphic presentation to be published in the United States.

In 1915 the American Society of Mechanical Engineers extended invitations to a number of associations of national scope to form a Joint Committee on Standards for Graphic Presentation. The purpose of the committee was expressed in the following terms:

"If simple and convenient standards can be found and made generally known, there will be possible a more universal use of graphic methods, with a consequent gain to mankind because of the greater speed and accuracy with which complex information may be imparted and interpreted." The report of the committee has contributed much towards standardization in the field of graphic presentation.

Graphic presentation has reached into all major fields since that time, and has evolved into universal usage.

Purpose of the Graph. The primary purpose of the graph is to present numerical data in visual form. With the growth of its use in numerous fields of endeavor, the functions of the graph have multiplied. It serves as a means of presenting visually tables of statistics in a simple, readable, and interesting form. The graph also makes clear undiscernible facts, such as correlations, which might be overlooked in tabulated data. It facilitates the presentation of facts for comparative purposes, and in many instances the graph indicates significant facts not obviously apparent in numerical form.

In some cases, where statistical information is available in tables, it is in the form of a complex arrangement of data. When it is difficult to make comparisons from such numerical data, however, a graphic presentation makes them available at a glance.

The most useful purpose of the graph is to save time and effort in analyzing statistics and tables.

The message delivered by the graph may be of a lasting or permanent nature. It is partly for this reason that the advertising profession has used graphs widely to convey information in advertisements.

The graph is used to portray the past, the present, and the probable future. Thus the graph is used for research purposes and historical comparisons, for analysis of current situations, and for forecasting the future. For example, in a business concern the graph is used not only for an analysis of current operations, the size of inventory, the personnel performance analysis, etc., but also in planning purchasing, production, flow of work, advertising, sales, marketing, departmental organization, etc. The graph is a convenient means of recording information for reference purposes. In addition, it permits the drawing of logical conclusions on the basis of the data depicted.

The mathematician finds the graph an invaluable aid in the solution of arithmetic and algebraic problems, the solution of mathematical formulas, and the representation of relationships.

The graph is now an essential tool for the student, the business executive, the educator, the banker, the biologist, the psychologist, the engineer, the sociologist, as well as for people in almost all other fields of endeavor.

Principle of the Graph. The basic form of the graph is derived by plotting figures in relation to two axes. These axes are formed by an intersection of two perpendicular lines. Through the use of scales of values along the axes, a means is provided for locating points.

A horizontal line, called the "X" axis (or abscissa), is drawn. A scale is arranged in both directions from a point (zero) at the center of the line. The scale divisions to the right of the zero point are designated positive values, while the divisions to the left of the line are designated negative values. A perpendicular is drawn through the zero point, and this vertical line is designated the "Y" axis (or ordinate). The scale divisions above the zero point are designated positive values, the divisions below negative values. The zero point, the point of intersection, is known as the point of origin. In the diagram, four areas known as quadrants, numbered counter-clockwise, have been formed as a result of the intersection.

The plotted points on most graphs are confined within Quadrant I. Quadrant II is seldom used in graphic presentation, while Quadrant III is rarely used, because two negative scales are involved.

Quadrants I and IV are occasionally used in presenting data, such as profits and losses, exports and imports, surplus and deficit, etc. (see Fig. 74, p. 108).

Given values, a corresponding point may be located on the

graph, or, given a point on the graph, its values may be determined. A point may be located on a graph (for example X=4, Y=3) in the following manner:

The horizontal axis (the X axis) is marked off in equal units, beginning with zero at the point of origin (intersection point). Since the given value for X is 4, a point is marked off at the fourth unit from zero on the X axis. The vertical axis likewise is

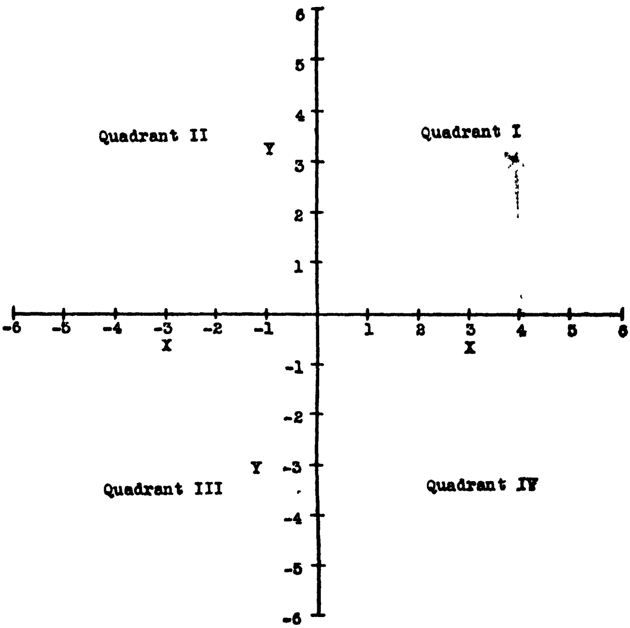


FIG. 1. THE BASIC FORM OF THE GRAPH

marked off into equal units, beginning with zero at the point of origin. The given Y value is 3, therefore a point is noted at the third unit from zero on the Y axis.

From each point indicated on each axis (X=4, Y=3) a perpendicular is erected. The perpendiculars will intersect to form a rectangle, as seen in Fig. 2 below. In such a manner the rectangular coordinates of a point are located. Thus the X coordinate is equal to 4, while the Y coordinate is 3. The intersection point

on the illustration (point a) is the location on the graph of the value 4, 3 or $X=4$, $Y=3$.

The method of determining the values of a point on a graph is given in the following example. From the point b, in Fig. 3, a line parallel to the X axis is drawn. The Y value of point b, or 5, is the value denoted at the intersection of this line and the Y ordinate. To obtain the X value, another line, parallel to the Y

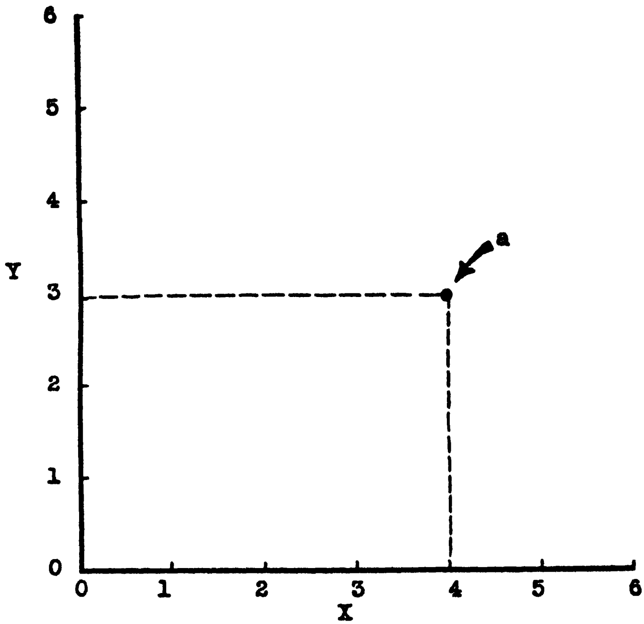


FIG. 2. LOCATION OF PLOTTED POINT $X=4$, $Y=3$ (QUADRANT I)

axis, is drawn from point b, and where this line meets the X axis, the X value is found (3 in this instance). A rectangle is formed as a result of the drawing of these lines. In such a manner the rectangular coordinates of a point are determined. Thus, one coordinate of point b is equal to 3, while the other coordinate is 5. The value of point b may be written as ($X=3$, $Y=5$). Most data used in the plotting of graphs lie in quadrant I where both ordinates and abscissae are positive.

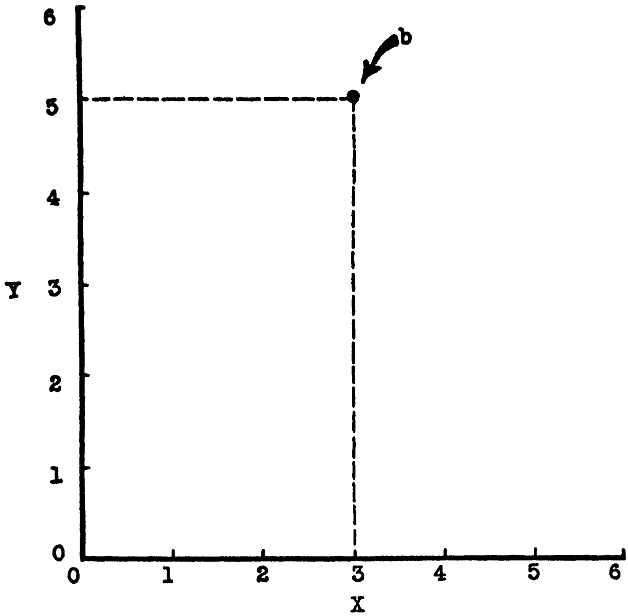


FIG. 3. DETERMINATION OF VALUES OF A POINT (QUADRANT I)

The practical application of the mathematical principles just noted may be seen in the following graph showing the production of passenger automobiles by years from 1928 to 1934.

TABLE I

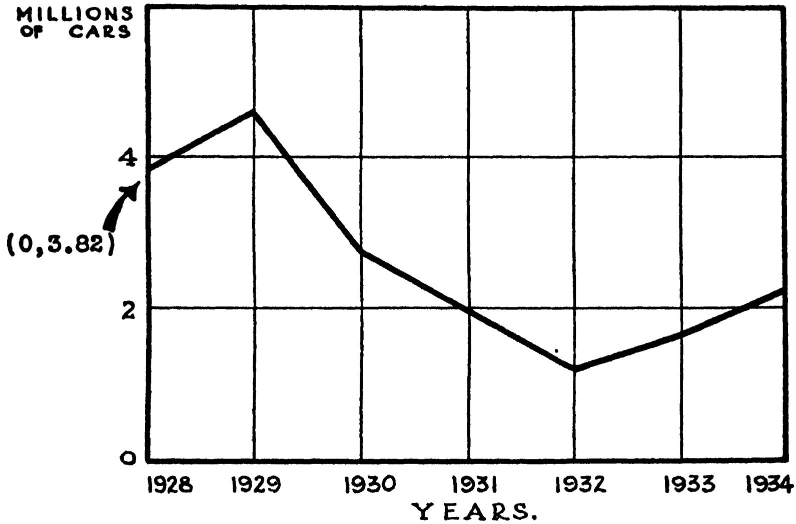
PRODUCTION OF PASSENGER AUTOMOBILES IN THE UNITED STATES
1928–1934

Year	Production Millions of Cars
1928	3.82
1929	4.59
1930	2.78
1931	1.97
1932	1.14
1933	1.61
1934	2.19

Source: United States Department of Commerce

Since both X and Y values are positive, Quadrant I forms the basis of the graph.

In plotting the two variables, years and production, the former is selected as the X axis, or independent variable, since changes in time are independent of changes in any other series. Therefore time is always used as an independent variable. Production which varies with time is placed on the Y axis.



Source: United States Department of Commerce

FIG. 4. PRODUCTION OF PASSENGER AUTOMOBILES IN THE UNITED STATES, 1928-1934 (LINE GRAPH)

On the X axis the zero point, or point of origin, will be 1928, since it is the starting-point of the series. From this point equal units are laid off on the X axis. Each unit on this axis will represent a year increase. On the Y axis a scale is constructed which will include all the production data to be plotted. In Chapter II the methods of constructing the graph and its component parts, as the scale, title, source, etc., are described in detail.

Since 1928 is at the point of origin or 0, and production at this point is 3.82 millions of cars, this point (0, 3.82) may be

plotted. The next point (1, 4.59), which represents production in 1929, or one year from the year of origin, as well as the rest of the points, are plotted in the same manner. The points are then connected by straight lines to form a continuous line.

For convenience in plotting, background lines are drawn parallel to the X axis at the scale divisions on the Y variable, and lines parallel to the Y axis at the scale divisions on the X axis. The resulting rectangles provide an excellent guide in plotting a point. These lines are drawn in pencil and later erased, leaving only absolutely necessary background lines to be inked.

CHAPTER II

CONSTRUCTION OF THE GRAPH

Types of Graphs—Elements of the Graph—The Grid—The Scale—Scale Captions—The Title—The Source—Accompanying Data—Key or Legend—Model Graph—Pictorial Graphs.

THE graph is a means of converting into visual form a comparison of various quantities, for various periods of time, for places, or for kinds of data.

This comparison is made by the variation in the height of a line, by the comparison of the lengths of bars, or by the comparison of the area or volume of geometric or irregular figures.

Types of Graphs. Thus, graphs may be classified according to the method of construction used for making the comparison.

A classification of the types of graphs is shown below:

A. Line graphs
B. Bar charts
C. Area diagrams
D. Solid diagrams
E. Statistical maps
F. Graphs of relationship
G. Graphs for computation

A. Variations in quantity in a line graph are represented by a line or curve which varies in its distance from the base line. A graph of this type is shown in Fig. 7, p. 19. The construction of the line type of graph is taken up in detail in Chapters V and VI.

B. The bar chart makes possible the visual comparison of quantities by representing the quantities as a series of bars of varying lengths. The bars are constructed so that their lengths are proportionate to the quantities which they represent. An example of a bar graph is shown in Fig. 65, p. 108. The various types of bar charts are discussed in Chapter VII.

C. The area diagram compares quantities by contrasting the areas of either geometric or irregular figures. An example of the area diagram is shown in Fig. 84, p. 130. The problems faced in constructing area diagrams are discussed in Chapter VIII.

D. The solid diagram is used to make comparisons by contrasting the size (volume) of solid figures. The solid diagram is discussed at length in Chapter VIII (see Fig. 87, p. 135).

E. Statistical maps visually compare quantities and also indicate location of the values (see Fig. 97, p. 151).

F. Graphs of relationship visualize the correlation between series of data (see Fig. 92, p. 142).

G. The graph may be used to facilitate calculations involving complex formulas where these computations are frequently repeated (see Fig. 127, p. 193).

Elements of the Graph. *The Grid.* The central portion of the graph, that section on which the bars, lines, etc., are actually drawn, is known as a background ruling or grid. The lines of the background ruling serve as an aid to the eye in making comparisons and facilitate the visual estimation of values. The grid should contain as few background lines as possible, since too many lines in the background ruling will tend to make the plotted data difficult to discern, especially in the line graph. Since the primary function of the graph is to facilitate visual comparisons, any practice which hinders the making of these comparisons should be avoided. An illustration of the confusion which results

from excess grid rulings is given in Fig. 5, in which there are illustrated a graph (Graph A) containing unnecessary background lines and the same graph (Graph B) containing the proper number of coordinate lines.

In order to facilitate plotting, many coordinate lines may be ruled lightly in pencil and erased subsequent to the inking in of the graph. The pencil lines may be erased by using a gum eraser or by washing the graph with benzine or carbon tetrachloride.

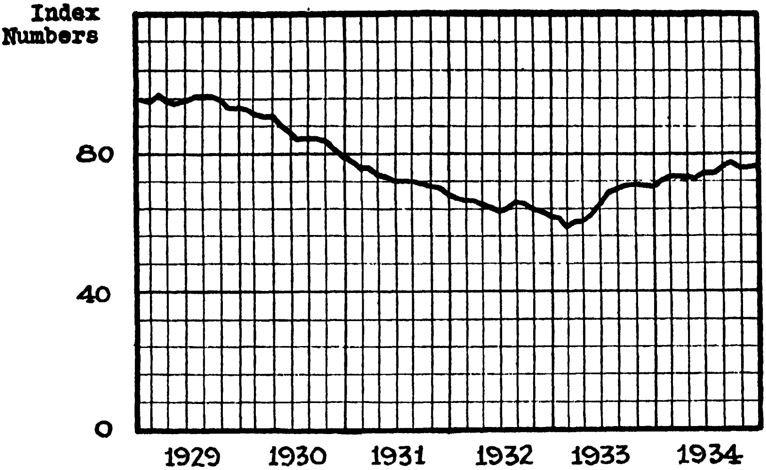
Numerous coordinate lines may be used on the grid if they are lightly ruled and of a color other than black. The printed sheets of graph paper are generally in light green or orange, against which the black of the curve lines stands out clearly. Samples of different types of graph paper are illustrated on pages 45 and 46 in Chapter IV.

The Scale. The rulings on the grid are evaluated through the use of scales of value which are placed along the horizontal and vertical axes. Since the X axis constitutes the bottom of the grid, the scale of values for this axis is placed along the bottom of the graph. The values on the scale progress from the lowest values at the left end of the grid to the highest at the right end.

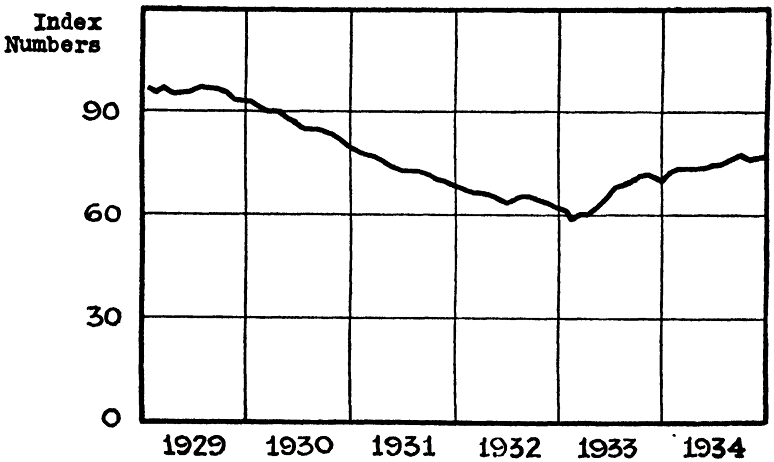
The scale of values for the vertical axis is placed at the left of the graph and runs from the lowest value at the bottom to the highest value at the top.

Few scale values should be used along the axes since only approximate and not exact values may be determined from a graph. Unnecessary scale values tend to hinder rather than facilitate the reading of plotted points on a graph. A large number of scale values may be inserted in pencil as a convenience in plotting, and later removed. A scale value should be placed next to each significant background ruling.

In preparing a scale of values, the first step is to determine the number of background lines (coordinates) it is desirable to use. This number is divided into the range of the values (the difference between the highest and lowest value to be plotted). The



Graph A. Too Many Coordinate Lines



Graph B. Proper Number of Coordinate Lines

Source: United States Bureau of Labor Statistics

FIG. 5. INDEX OF WHOLESALE PRICES IN THE UNITED STATES (1926 = 100)
(ILLUSTRATING PROPER NUMBER OF COORDINATE LINES)

resulting figure should be rounded off to the next highest round figure, and used as the difference between the successively indicated scale values.

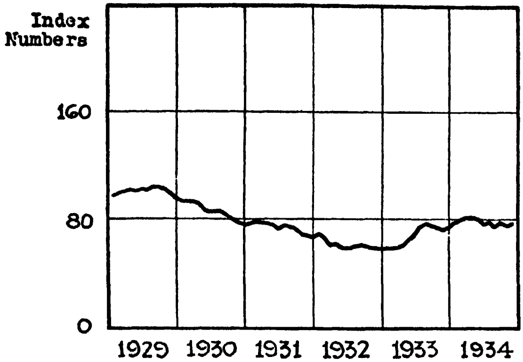
Thus, having been given the figures in Table I on page 7, four coordinate lines were selected as the most desirable number. The data ranged from 1.14 million cars to 4.59 million cars, but since the zero value must be included, the range used is from 0 to 4.59 million cars, or 4.59. This number was then divided by 3, the number of desired coordinate lines. The resulting figure was rounded off to 2 (see p. 49 for the principles of "rounding off" figures) and the scale values fixed with 2 as the difference between scale values.

It is important to note that the size of the grid used, and the range of values decided upon, will determine the degree of variation exhibited by the graph. Thus, if the vertical scale is increased in length in relation to the horizontal axis, the variations occurring will be emphasized. If its length is contracted, the fluctuations will tend to be reduced. In general, a graph with a ratio of width to height of between 1.25 and 1.75 to 1 seems to give the best results.

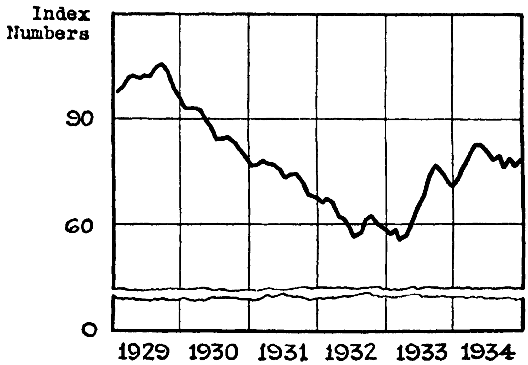
An example of the distortion occasioned by the incorrect proportioning of the horizontal scale to the vertical scale is illustrated in Fig. 6 (a and b), showing factory employment in the United States, 1929-1934. The same graph with a satisfactory relationship between axes is shown in Fig. 6 c.

The grid and scale should be so planned that the plotted data will require the use of almost all of the scale. It is advisable, where possible, to have some space on the grid, both above and below the plotted values.

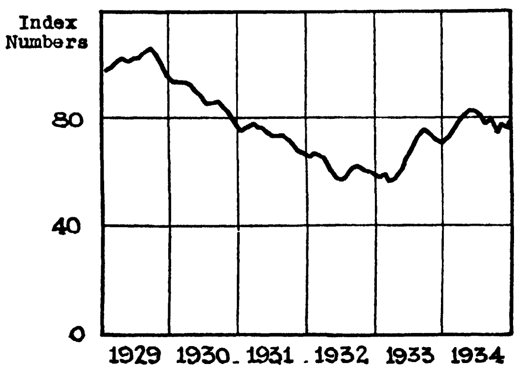
Scale Captions. Scale captions must be placed on both axes to clearly identify scale values. It is essential that both the subject of the scale, and the units used, be indicated, viz., Sales in Thousands of Dollars. The caption for the horizontal scale is



Graph A



Graph B



Graph C

Source: Federal Reserve Board

FIG. 6. FACTORY EMPLOYMENT IN THE UNITED STATES, 1929-1934 (1923-1925 = 100) (VARIOUS SCALES)

generally placed under the horizontal axis at the center.[1] The caption for the vertical axis is best placed at the top left of the axis. The words or letters of the caption are printed parallel to the base of the graph so that the paper need not be turned to read the label. This practice is not always followed, and as a result graphs are sometimes difficult to read. The correct positions for scale captions may be seen in Fig. 7 on p. 19.

The Title. Graphs are frequently examined by those who do not read any accompanying textual explanation, even if such an explanation is readily available. Therefore, a graph should be understandable on the basis of the information given by it alone, and for this reason the title must be complete.

The title should give the "what," "where," and "when" of the data, in the order indicated. Titles may be used in the "complete" form or the necessary information may be conveyed by means of subtitles. The latter method has the advantage of greater attention attracting value.

An example of the two types of titles is given below:

Index of Stocks of Leather in
the United States, 1923–1934
1923–1925 = 100

or

Leather Stocks
Index for the United States, 1923–1934
1923–1925 = 100

The position of the title is determined by the use of the graph. In publications, especially in books, it is the general practice to place the title beneath the graph with an identifying Fig. number. In all other work and, upon occasion, in magazines or pamphlets where the attention attracting value of an outstanding title is desired, the title is centered at the top of the graph. In this position it is of predominant importance. When placed at the top of the graph the title is generally printed in larger letters than

[1] An exception to this rule is found in one type of bar chart. See p. 116.

any other lettering on it. Subtitles in some instances are printed in slightly smaller letters than the title.

The Source. It is always desirable to indicate on the graph the source of the data used to construct it. This is done for two purposes: (1) to lend the authoritativeness of the original source to the message presented, and (2) to enable the reader to obtain further data if he so desires.

The source is generally placed immediately beneath the graph to the lower left-hand side (see Fig. 7). It may be as complete as is desired, but should at least give the name of the compiling or publishing agency. If desired, the entire name of the publication from which the data were taken may be used, and the page reference as well. Various practices in indicating sources according to the completeness deemed necessary may be seen in Fig. 89, p. 137; Fig. 32, p. 55; and Fig. 36, p. 58.

If the data are original, no source is necessary. If the entire graph is reproduced, the source may be indicated in the following form:

From *Electrical Railway Journal,* September, 1931 (see Fig. 91, p. 141), or Reproduced from "Agricultural Regions of Asia," Cressy, G. B., *Economic Geography,* April, 1934 (see Fig. 104).

Accompanying Data. Although at one time it was considered good form to include on the face of the graph the data from which it was constructed, common practice now dictates that the graph be accompanied with a table of data.

Since it is not intended that the reader be able to obtain the original figures from the graph with any degree of accuracy, it is always advisable to include such a table. For the rules covering the types and construction of tables see Chapter XV.

Key or Legend. When several variables are included on the same graph it is necessary to identify each by using a key or legend.

A sample of each type of line, color, shading, or cross-hatching used for distinguishing purposes is indicated and identified. This identifying key or legend is generally enclosed within a box and, where possible, placed within the grid itself. If placed in this position care must be taken to see that the key does not interfere with curve lines. The legend should not be placed against the border of the graph.

If the entire face of the graph should be fully occupied, as in the case of some types of bar charts, the key may be placed immediately beneath the grid, in which case it is not generally enclosed in a box. An example of the legend placed outside of the grid may be seen in Fig. 72, p. 118.

Model Graph. The proper placing of the elements of the graph is shown in Fig. 7. The title is shown in the position for an unpublished chart.

The following characteristics of parts of the model graph should be noted:

1. The completeness of the title, explaining in order what, where, when.
2. The relative size of the lettering in the title, scale captions, etc.
3. The location of the scale captions.
4. The "round" values on the scale.
5. The emphasized curve lines.
6. The order of years, from the earliest to the latest year.
7. The source—in this case merely the compiling agency was indicated.

Pictorial Graphs. Pictures serve as an aid in attracting attention to the graph and in heightening interest in the subject-matter therein presented. Frequently they convey the description of the data more quickly and effectively than a lengthy title, and always serve to enliven the graph.

Pictures may be used in a number of ways as an aid to graphic presentation. The picture itself may present the comparison as a form of area diagram, solid diagram, or multiple unit bar chart (see Figs. 84, 88, and 90), the picture may be superimposed on a

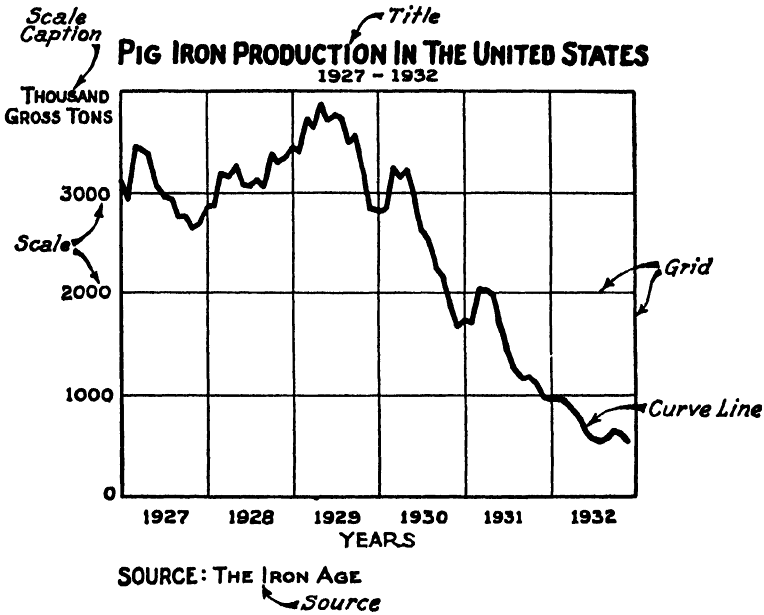


FIG. 7. MODEL GRAPH

bar chart or area diagram (see Fig. 75), or it may be used as a background for the graph. The background in this type of graph serves to imprint upon the reader's mind the subject dealt with, in addition to attracting attention due to its unusual nature.

CHAPTER III

GRAPHIC LAYOUT

GRAPHS are used for a wide variety of purposes. Depending upon the use for which it is prepared a graph may appear in one of a number of forms and in different sizes. Graphs may be prepared in the form of large display charts to be shown to groups of people in order to illustrate or emphasize an oral message. The graph may be carefully prepared in a small-sized form if it is to be shown to a few people individually, bound into a report, or if intended for printed publication in book or pamphlet form. When it is prepared for the personal use of an individual for purposes of analysis, the graph may be presented in a less formal form. However, in whatever form it is to appear, the same general principles of graphic presentation apply and uniform care in planning and design is essential.

A graph that is poorly planned and executed will defeat its own purpose. Its message will not be readily understood. Thus, the graph will fail to attract the required attention of the reader, resulting in a general lack of confidence in the reliability of the message presented.

An effectively prepared graph should accomplish much the same functions as an advertisement.

1. It should attract the attention of the reader.
2. It should deliver its message quickly and simply.
3. It should be completely self-explanatory and readily understood.
4. It should inspire confidence in the reader by its workmanlike construction.

In order to accomplish these functions efficiently, considerable advance planning is necessary. The most effective graph will result if, prior to the actual construction of the graph, a "layout," or rough sketch of the graph as it will appear when completed, is prepared. This layout serves a two-fold purpose. It secures the most effective design and most satisfactory proportions

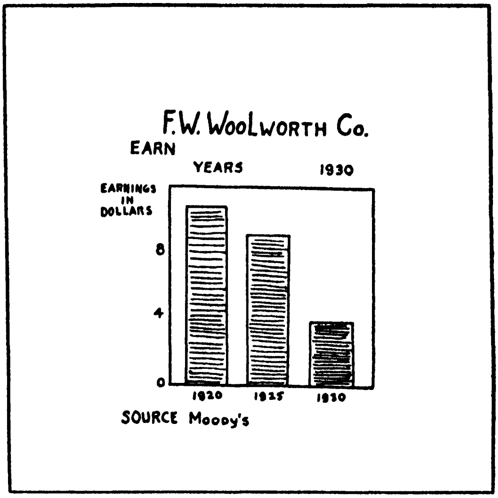


Fig. 7a. Layout for Graph on Page 108

for the resulting graph. It may be used as a set of directions for the person actually constructing the graph. A layout in preliminary form for the graph shown in Fig. 65 is illustrated in Fig. 7a.

In preparing the layout as a guide to the construction of the graph, it is necessary to consider a number of important points in graphic design including:

1. The size of the graph.
2. The proportions of the graph.
3. The size and prominence of the title.
4. The emphasis of the contrast between the data shown in the chart.

5. The ability of the graph to attract the attention of the reader.

6. A workmanlike and pleasing appearance for the graph.

The size of the graph is closely related to the purpose for which the graph is to be used. It will be large if it is to be displayed to a large group of people, smaller if it is to be shown to but a few persons. It may be of comparatively small size if intended for a single individual. Thus, the size of the chart depends largely upon its legibility to the reader and the cir-

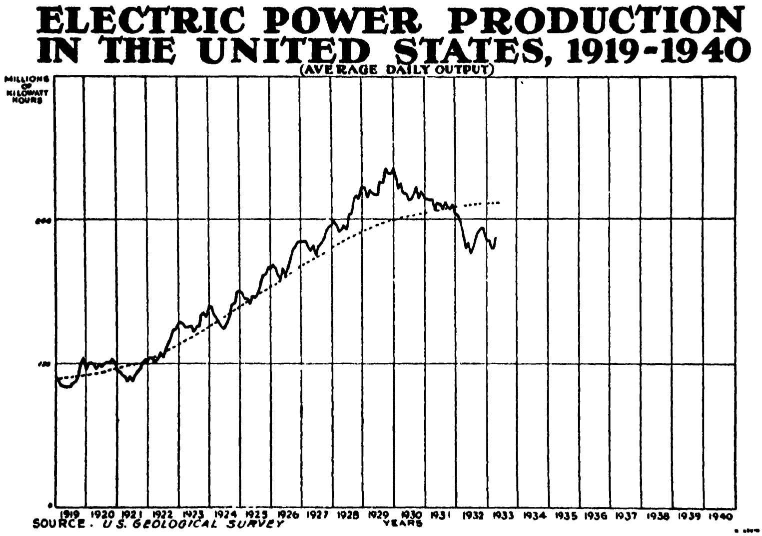
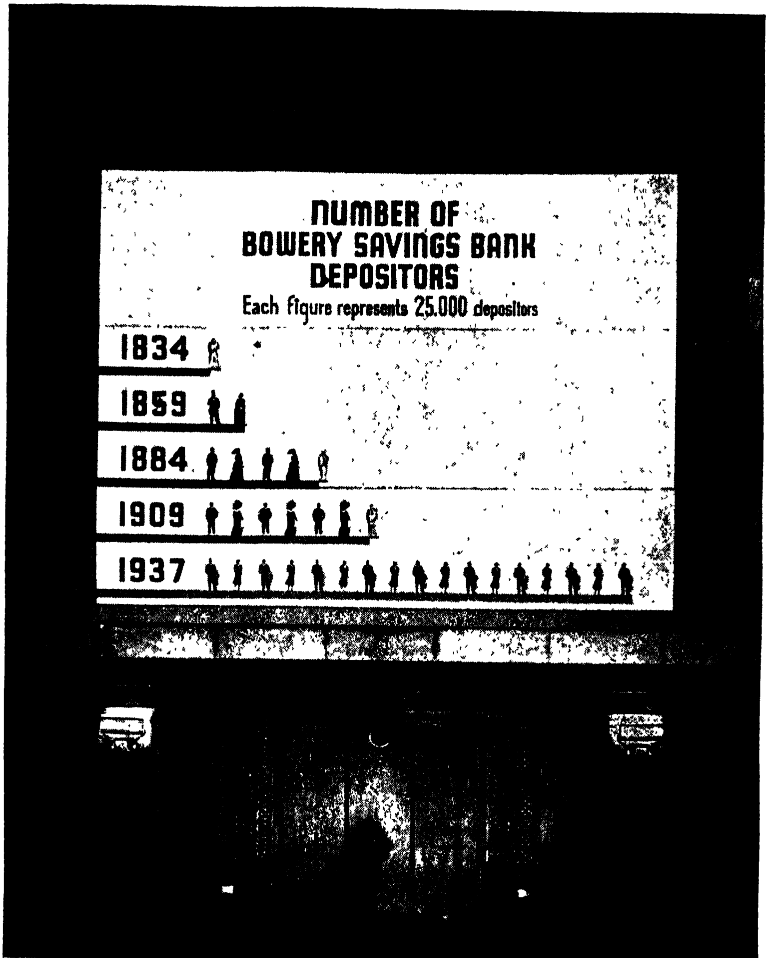


FIG. 7b. A GRAPH PREPARED FOR DISPLAY PURPOSES

cumstances under which he will examine it. Generally, the graph should be of the minimum size which will effectively gain that legibility.

The general proportions of the finished graph should be pleasing to the eye. Graphs prepared on the standard sized sheets, 8½″x11″, 11″x17″, 15″x22″, 22″x30″, etc., are usually of efficient proportions when properly fitted to the page.

The graph should be placed upon the sheet in such a way as to make it stand out clearly. This may be accomplished by using a



Courtesy of the Bowery Savings Bank, New York City

FIG. 7c. A GRAPH USED AS A MURAL AT THE BOWERY SAVINGS BANK, NEW YORK CITY

suitable margin on all sides of the paper, thereby providing sufficient surrounding white space.

The lettering of a graph should be clearly legible at the required reading distance. Thus the size of the letters must be gauged accordingly, with the title assigned a predominant size. The title should be lettered in a size somewhat larger than that of the other lettering on the chart. The source, scale captions, etc. should be lettered in the minimum legible sizes.

The curve lines, bars or other forms used to contrast the quantities to be compared in a graph should be emphasized. When areas, as within bars, pie diagrams, etc., are used, they should be filled in with a solid color or with cross hatching to make them stand out clearly against the background. The silhouettes created by cross hatched, or filled in areas add to the attraction value of the graph.

The title on the graph must be used for approximately the same purpose as the headline in a newspaper i.e., to catch the attention of the reader and arouse his interest in the details of the story to be presented. The attraction value of a graph in large part will depend upon care in the wording of the title. Therefore excess verbiage should be avoided.

A further attention attraction value can be secured for any graph through the use of colors, especially vivid colors. However, resort to the use of colors for this purpose renders the problem of reproduction more difficult and expensive.[1]

[1] See Chapter XIV for the methods of reproducing graphs.

CHAPTER IV

THE EQUIPMENT FOR GRAPHIC PRESENTATION

Equipment—Guides—Drawing Board—Mounting the Paper—T Square—Triangle—French Curve—Flexible Curve Ruler—Drawing Instruments—Ruling Pen—Compass—Measuring Instruments—Rulers—Scales—Flat Scale—Triangular Scale—Dividers—Protractor—Lettering—Single Stroke Lettering; Built-up Lettering—Lettering Equipment—Lettering Pens—Mechanical Lettering Equipment—Paper—Ink—Printed Graph Sheets.

THE effectiveness of a graph is largely dependent upon its presentation. The degree of authoritativeness with which a graph is endowed is closely related to its physical appearance.

Since the appearance of a graph plays such a large part in determining its effectiveness, much attention must be paid to the physical details of its construction. It is attention to detail and a knowledge of the equipment which may be used to obtain a desired result that distinguishes the workmanlike graph. A properly prepared graph, therefore, is not of necessity the result of long training or experience.

Equipment. The equipment used in the construction of a graph need not be elaborate. Its complexity will depend on the quantity of work required and the variety of graphs to be constructed.

It is possible to construct a graph with merely a ruler, a pen, and India ink. However, the minimum amount of equipment that will give satisfactory results for even a simple graph con-

sists of a drawing board, T square, triangle, ruling pen, India ink, and lettering pen.

The drafting equipment used in graphic construction may be classified as follows:

1. Guides (for ruling lines and curves).
2. Drawing instruments.
3. Measuring instruments.
4. Lettering equipment.

Guides. *Drawing Board.* The first step in preparing a graph is to mount a suitable sheet of paper on a *drawing board.* This board, upon which all of the work is done, is generally made of white pine and ranges in size from 12x17 inches to 31x42 inches, or larger.

Mounting the Paper. The paper is mounted on the drawing board by means of thumb tacks. The tacks are pushed through the corners of the paper into the board, or driven into the board close to the edges of the paper so that the heads of the tacks will hold the paper firmly in position. Although the first method is commonly used in drafting, the paper must be trimmed to eliminate the holes made by the thumb tacks. If the paper is of a standard size which must be retained, as is common practice in statistical work, trimming is not possible. The second method eliminates the placing of holes in the paper.

Scotch tape, an adhesive paper tape, may be used to fasten the paper to the board. The tape may later be removed without injury to the drawing. The use of Scotch tape is preferable to the thumb-tack method, since continued use of thumb tacks and the resulting holes in the board, reduces the serviceability of the board and in time may render it useless.[1]

When the paper is placed on the drawing board its edges should be aligned so that the edges of the sheet will be parallel to the edges of the drawing board. The T square (a picture of

[1] This may be avoided if the tacks are placed in holes already made in the board.

which is seen in Fig. 8) aids in aligning the paper. When the paper is properly aligned, a line drawn along the edge of the T square will be parallel to the edges of the sheet of paper.

T squares may be obtained in a number of forms, one of the best of which is the type with a transparent xylonite edge. The T square is used to draw horizontal lines.

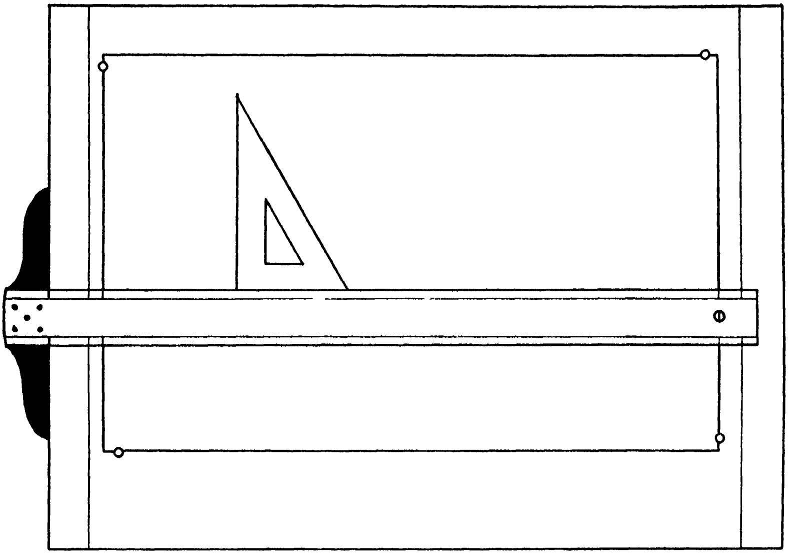
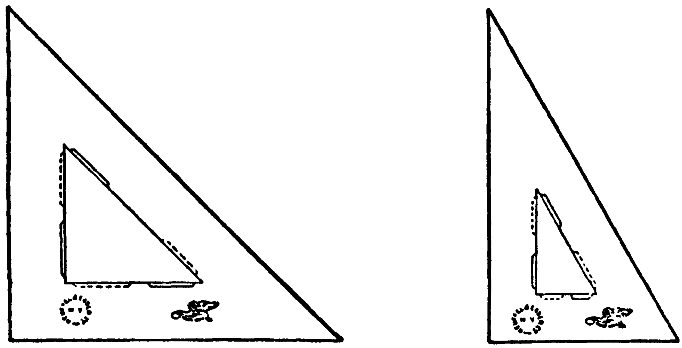


FIG. 8. DRAWING BOARD, T SQUARE, AND TRIANGLE

To draw a perpendicular to the line drawn along the T square, a triangle may be used. If the triangle is rested against the side of the T square and a line is drawn along its side, the resulting line will be parallel to the edges of the sheet.

Triangles may be obtained in various types and of different materials. The commonly used types are the 30°x60°x90° and 45°x45°x90° triangles. The triangles are made of many materials, such as wood, hard rubber, and xylonite. Xylonite is the best type for general purposes, since it is transparent and thus does not obscure part of the drawing while being used.

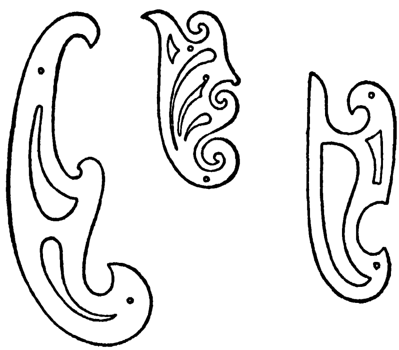
In drawing curved lines, either *French curves* or *flexible rulers* may be used. French curves come in a wide variety of shapes, some of which are shown in Fig. 10.



Courtesy of Keuffel and Esser Co.

FIG. 9. XYLONITE TRIANGLES

The curve is first drawn roughly in pencil. The French curve is then adjusted until one of its various sides matches the section of the curve to be inked. It is then shifted to the next section of



Courtesy of Keuffel and Esser Co.

FIG. 10. FRENCH CURVES

the curve and again adjusted to match. This is continued until the entire curve has been completed.

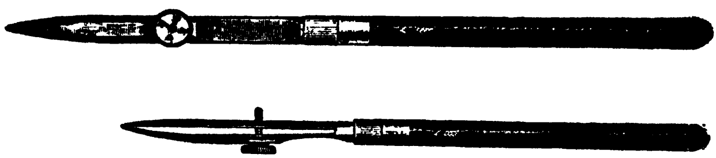
If any number of various types of curves are to be drawn a

number of French curves may have to be tried in each case before the right one is found.

Although the flexible ruler is less commonly used, it is especially adaptable for statistical drafting. It is so constructed that it can be bent into any desired curve shape.

Work may be considerably expedited if a guide, such as a triangle, is rested on another guide, such as another triangle or a T square, thus raising it above the surface of the drawing. If this is done there will be no danger of smearing wet lines, and it will not be necessary to wait for the drawn lines to dry.

Drawing Instruments. The most useful drawing instrument in preparing graphs is the *ruling pen.* It may be used to draw lines of any desired width by adjusting the thumbscrew to widen or narrow the distance between the prongs (see Fig. 11).



Courtesy of Keuffel and Esser Co.

FIG. 11. RULING PEN

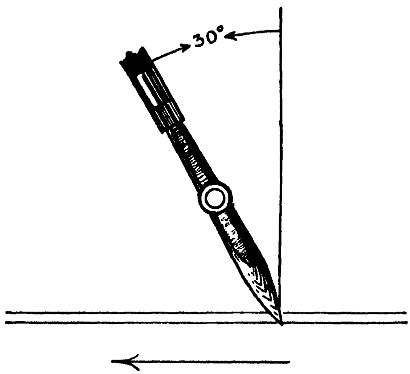
The pen is prepared for use by adjusting the prongs until there is a moderate distance between them. Ink is then placed between the prongs by inserting the quill attached to the cork of the India-ink bottle. *The ruling pen should never be dipped in the ink.*[2] This process may be continued until a sufficient quantity of ink has accumulated between the prongs. A little experience will soon indicate the correct quantity. Too little ink will cause the pen to run dry in the middle of a line.

The ruling pen should be held upward, but inclined in the

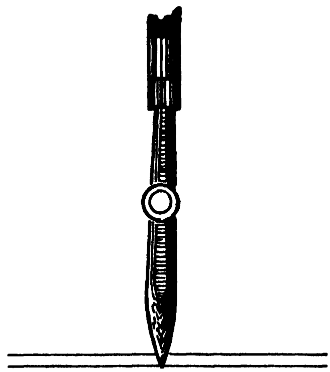
[2] If this is done ink will remain on the sides of the pen. In wiping off the ink from the sides, the ink between the prongs will also be removed.

direction of the motion. The pen should be held with the thumbscrew on the outer side and between the middle finger and thumb. The shaft of the pen should rest against the index finger near the first knuckle. The side of the small finger will then rest on the guide. The pen should then be drawn along the guide with a steady smooth motion and always held at the same angle. The correct and incorrect positions of holding the pen are shown in Figs. 12 and 13.

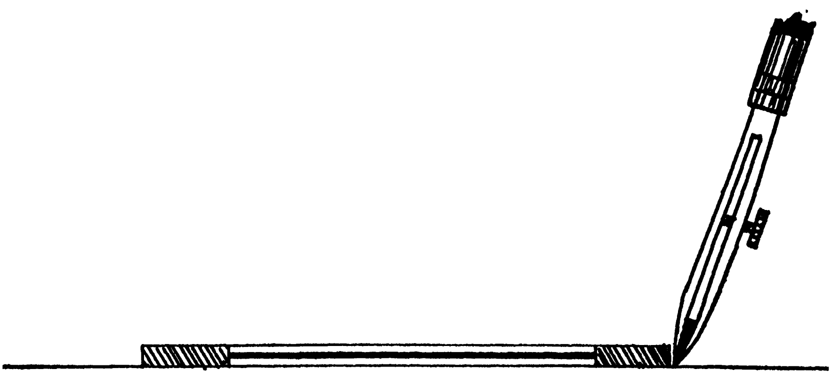
The pen must be kept clean and wiped of ink after it is used, in order to prevent corrosion. A corroded pen will not draw a smooth line.



Correct Position

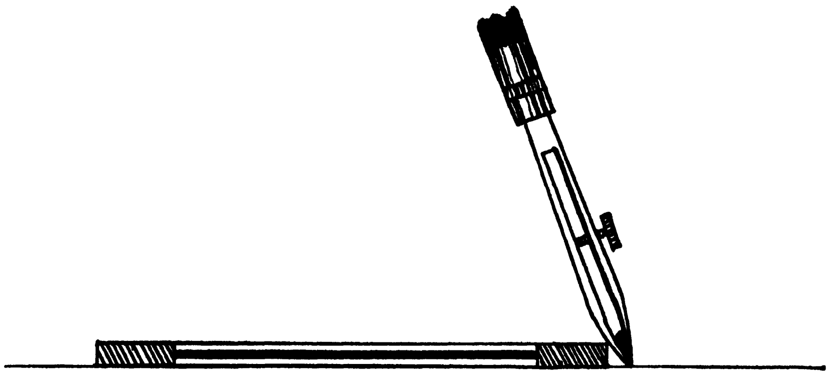


Incorrect Position

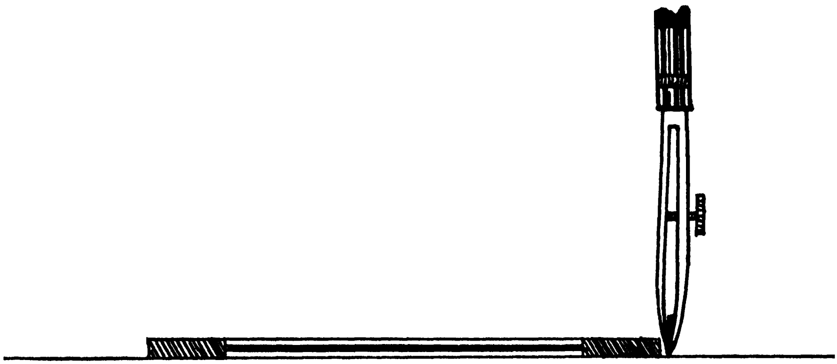


Incorrect Position

FIG. 12. POSITIONS OF THE RULING PEN



Incorrect Position



Correct Position

FIG. 12. CONTINUED

For heavy lines in larger graphs the detail pen may be used. A detail pen is pictured in Fig. 14.

It is always advisable to construct the entire graph in pencil before inking in any part of it. The penciled lines may later be removed by means of a gum eraser, benzine, or carbon-tetrachloride.

The *compass*, another valuable form of drawing instrument, is used to draw circles. The compass is generally provided with both a pencil and ink attachment which are interchangeable. The ink attachment is of the nature of a miniature ruling pen. One of the

shafts of the compass is fitted with a needle point, which is placed at the center of the projected circle and the other arm is revolved

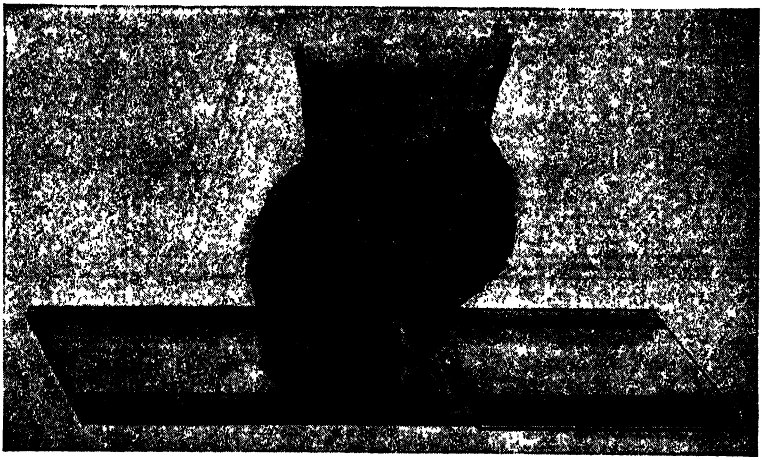
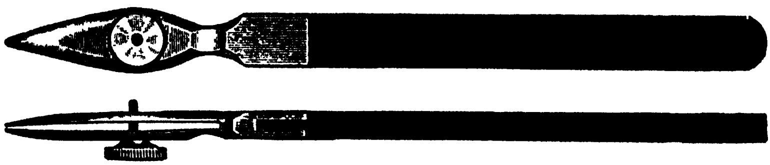


FIG. 13. THE CORRECT POSITION OF THE RULING PEN

about it. The arms of the compass usually have a pivot by means of which the bottom part of the arms can be kept perpendicular



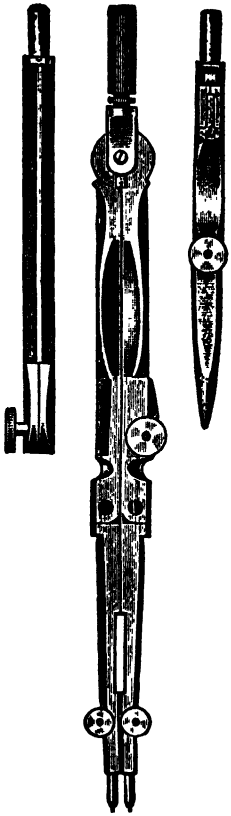
Courtesy of Keuffel and Esser Co.

FIG. 14. THE DETAIL PEN

to the paper. An illustration of the compass with pen and pencil attachments is seen below in the illustration.

Measuring Instruments. The instruments which are used for dividing a given space into a given number of parts, or for measuring, may be classified as follows:

1. Rulers.
2. Scales.
3. Dividers.
4. Protractors.



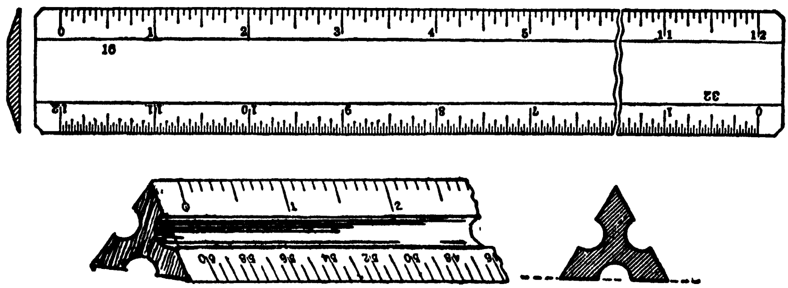
Courtesy of Keuffel and Esser Co.

FIG. 15. COMPASS WITH PENCIL, PEN, AND EXTENSION BAR ATTACHMENTS

The *ruler* is the simplest and most commonly used measuring instrument. It serves as a straight-edge as well as a measuring instrument. However, since only one scale of measuring units appears on the ruler, generally 6, 12, or 18 inches divided into eighths or sixteenths, it is not so flexible as the scale. An addi-

tional disadvantage is that the scale of values on the ruler is seldom accurately constructed.

The *scale* is the instrument which is generally used by draftsmen for measuring. There are two types of scales, the *flat scale*, with one, two, or four scales of values, and the *triangular scale*, with six scales of values.



Courtesy of Keuffel and Esser Co.

FIG. 16. FLAT AND TRIANGULAR SCALES

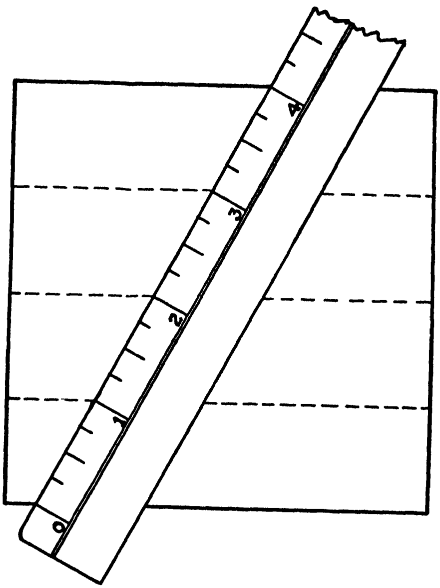
These scales, classified according to the type of divisions appearing on them, are as follows:

1. Architect's scale
   With scales of 1/64, 1/32, 1/16, 1/8, 1/4, 1/2, of an inch, or multiples of these.
2. Engineer's scale
   1/10, 1/20, 1/30, 1/40,
   1/50, 1/60 of an inch.
3. Statistician's scale
   One face with 3 logarithmic scales and the others in 1/20, 1/30, 1/40, 1/50, 1/60, of an inch.

The scales are generally 6 to 12 inches in length.

A given distance may be divided into a given number of parts by the use of a ruler or scale with almost any scale division. The method of dividing a space into equal divisions without a satisfactory scale—*i.e.,* one equally divided into the required divisions—is as follows:

If a space such as that in Fig. 17 is to be divided into four parts and no adequate scale is available, an ordinary ruler may be used. Four consecutive units on the ruler, such as four inches in this instance, are determined. The ruler is adjusted until the beginning of the scale intersects the bottom line while the end of the desired number of units on the ruler intersects the top line. Points



Courtesy of Keuffel and Esser Co.

FIG. 17. DIVIDING A GIVEN SPACE INTO A DESIRED NUMBER OF PARTS

are marked off at each of the four units and horizontal lines drawn through these points. Thus the distance has been divided into the desired number of parts, in this instance, four.

A more flexible means of dividing lines and marking off distances is provided through the use of the *divider*. The *divider* (see Fig. 18) is adjusted to an approximation of the desired division and is spaced off along the line to be divided. If the spacing falls short of the required distance, the space between the arms is increased; if it is more than the required distance, the distance be-

tween the arms is decreased. A measurement is made by holding fixed the position of the forward arm of the divider and advancing the other arm, repeating this process the required number of times.

Courtesy of Keuffel and Esser Co.

Large Dividers	Hairspring Dividers	Steelspring Bow Dividers	Ringhead Bow Dividers

Fig. 18. Dividers

The types of dividers may be classified as follows:

1. Plain dividers.
2. Hairspring dividers.
3. Bow dividers.

The *plain divider* (see Fig. 18) is generally used for the larger measurements.

The *hairspring divider* (see Fig. 18) is a plain divider with a fine adjustment in one of its shafts controlled by a thumbscrew.

The *bow divider* is used to make smaller divisions. The distance between the shafts is controlled by a thumbscrew, and they are held firmly in position by a spring. The steelspring and ringhead

dividers, two different types of bow dividers, are illustrated in Fig. 18.

The *protractor* is used to measure angles and is of particular value in the preparation of *pie diagrams*. It consists of a half-circle, usually of steel or celluloid, with its rim divided into 180 equal parts. These parts are called degrees of the arc. In order to measure an angle with the aid of the protractor, the instrument is laid on the angle in such a way that its center is directly over the vertex of the angle and zero on the scale is over one side of the angle. The number of degrees of the angle is indicated at that point on the protractor where the other side of the angle crosses the scale.

Lettering. The effectiveness of a graph is largely dependent upon its appearance, while the most important factors in its appearance are the neatness and legibility of its lettering. Skill in lettering is acquired through practice and through the use of the proper equipment.

In small charts the typewriter plays an invaluable part in quick, legible, and neat lettering. For those unskilled in lettering by hand, this method of lettering a chart is probably the most effective. The major objection to the use of the typewriter is the fixed size and style of type. The objection can be overcome to a degree by the use of the Variatyper, a typewriter in which a large variety of different kinds of type may be placed.

Lettering may be grouped into two general classes, single-stroke and built-up lettering. Single-stroke lettering is constructed by a single stroke of the pen. Each part of the letter is only as thick as the line made by the pen. Single-stroke lettering is generally freehand, but may be constructed with the aid of mechanical lettering aids, such as templates or guides. Built-up lettering is constructed with the aid of ruling pen, straight-edge, and compass. The outlines of the letters are drawn with the aid of these

instruments, and the inner parts of the letters are then filled in by pen or brush.

Single-stroke Lettering. Single-stroke lettering is most effective for lettering on small charts and for constructing small letters on large charts.

Single-stroke lettering is based on straight lines and circles. Each letter is constructed of straight lines or circles or of a combination of the two. Guide lines are ruled in with the aid of a lettering triangle or lettering instrument. For this purpose three parallel horizontal lines are drawn with the middle line spaced one-third of the distance away from the upper line. The height of upper-case (capital) letters is determined by the distance between the two outer lines, while the lower-case (small) letters are lettered between the two lower lines.

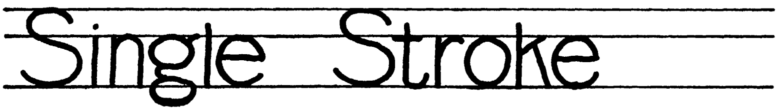


FIG. 19. GUIDE LINES FOR SINGLE STROKE LETTERS

The use of a lettering triangle or lettering instrument saves considerable time in preparing guide lines. The lettering triangle is placed against a straight-edge. A *soft* pencil with a long sharp point is placed so that the point passes through the proper holes for any desired size of letters. By sliding the triangle quickly along the straight-edge the appropriate guide lines are drawn.

In preparing lettering, the original work should always be done in pencil. A soft pencil should be used for this purpose, since a hard pencil may leave tracks in the paper which cannot be removed easily. By outlining the lettering in pencil, proper spacing can be planned.

Letters composed into words are not spaced at a uniform distance, but are so arranged that there are equal *areas* of background between each letter. Uniform distances between letters

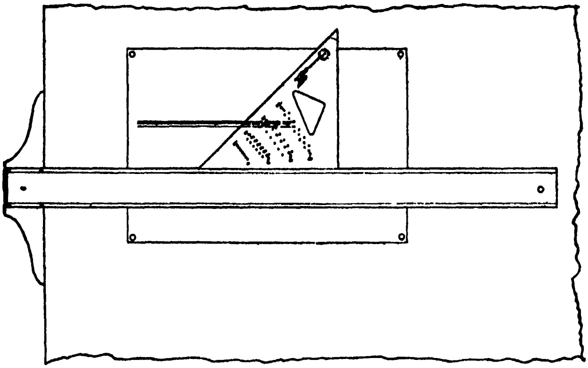


FIG. 20. THE LETTERING TRIANGLE

will not give the appearance of a uniform word because of the optical effect given by the background areas. Correct and incorrect spacings are illustrated in Fig. 21.

(Incorrect)

(Correct)

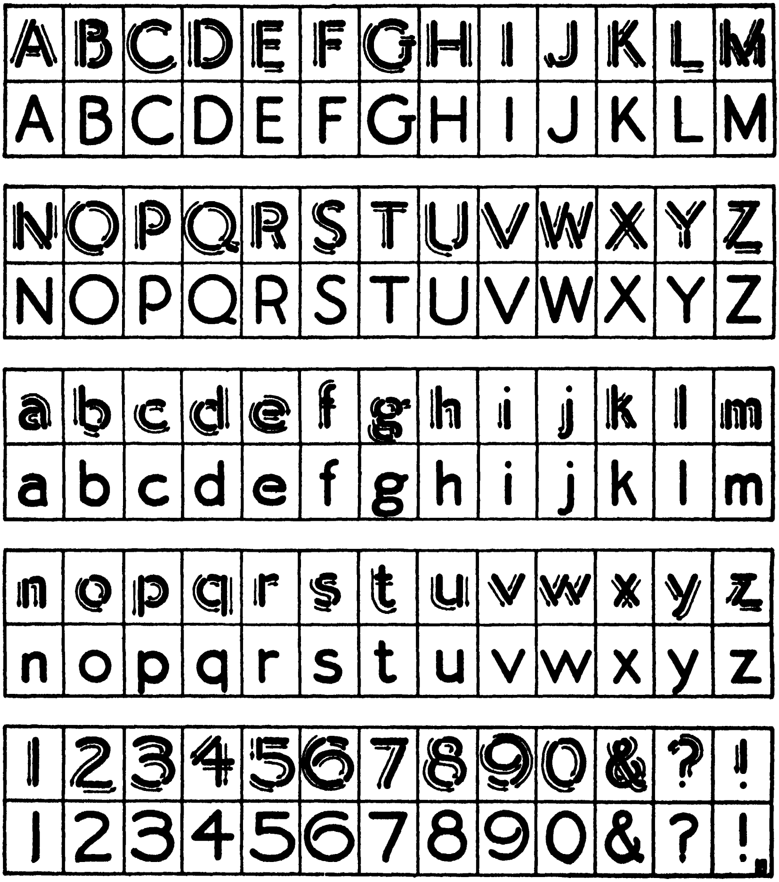
FIG. 21. SPACING THE LETTERS

Thus, the spacing between the letters I and N will be greater than the spacing between E and T because the shape of the letters does not permit of as much background area between them as when they are spaced evenly. A glance at Fig. 21 will show that when the spacing between the letters in this word is equal, the end of the word seems crowded.

Very effective lettering can be attained by the single-stroke method after some practice in spacing and construction. Practice books are available for this purpose. In Fig. 22 there is shown, through stroke analysis, the technique for drawing single-stroke letters.

Built-up Lettering. Built-up lettering is used in the preparation of lettering for large charts and for large lettering (such as titles)

on small graphs. In this type of lettering the outlines of the letter are prepared with the use of a ruling pen, straight-edge, and compass. The inner part of the letter may then be filled in by means

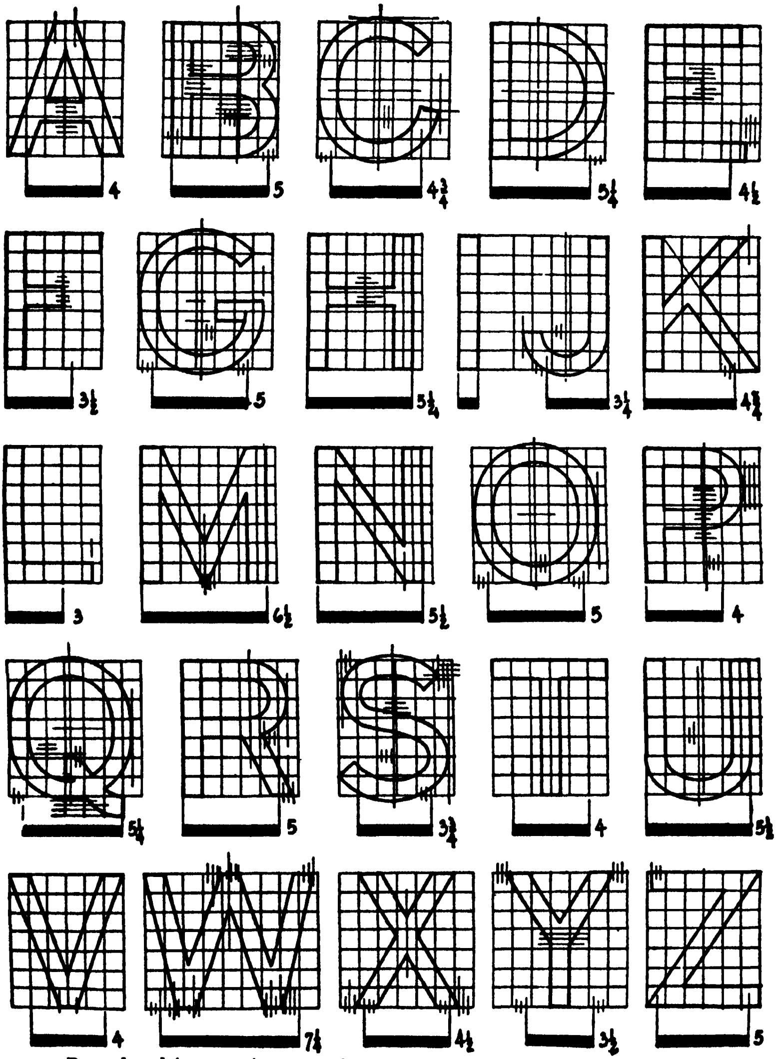


Reproduced from Weiss, E., "The Design of Lettering," Pencil Points Press, New York, 1932

FIG. 22. STROKE ANALYSIS FOR SINGLE STROKE LETTERS

of a Speedball pen, or by using a brush. The construction of simple block letters is shown in Fig. 23.

Examples of more elaborate types of lettering are shown in Fig. 23. It must be remembered, however, that the simpler forms



Reproduced from Weiss, E., "The Design of Lettering," Pencil Points Press, New York, 1932

FIG. 23. CONSTRUCTION OF BLOCK LETTERS

of lettering lend greatly to visibility and are much easier to construct.

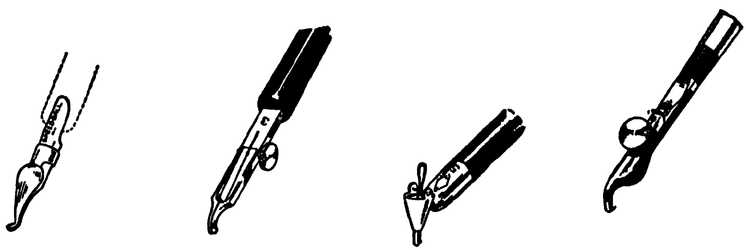
State **The New**

Block Letter Retail

Imported **Printers**

FIG. 24. TYPES OF BUILT UP LETTERING

Lettering Equipment. There are a number of pens specially constructed for lettering. The Speedball, Barch-Payzant, Perfection, and Shepard are the better known pens of this type. They are available in a number of graded sizes for lettering any size of letter. Examples of each of these types of lettering pens may be seen in Fig. 25 below.



Courtesy of Keuffel and Esser Co.

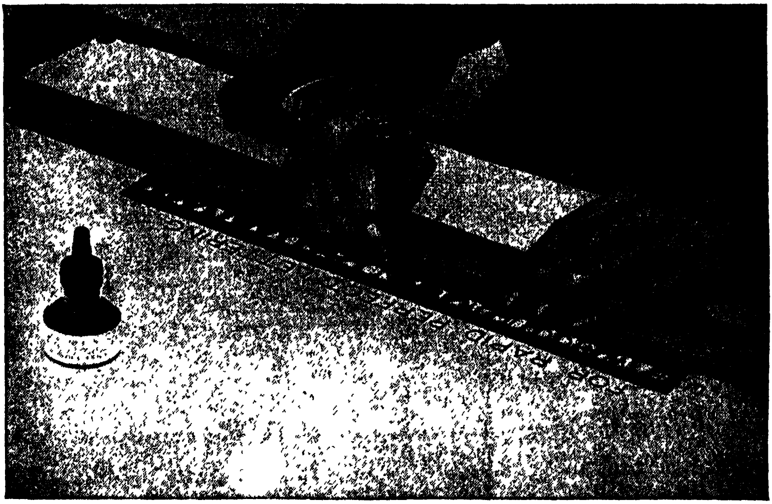
Speedball Pen Barch-Payzant Pen Perfection Pen Shepard Pen

FIG. 25. LETTERING PENS

For small lettering, a Spencerian J pen, a Gillot No. 105, an Esterbrook drawlet, a small-size Speedball, a Hunt No. 512, a

Tank No. 4, and many other similar pens are useful. Individual differences, however, will determine the best pen for a particular person.

A new pen should always be moistened before using to remove the film of oil on it. A well-broken-in lettering pen is more serviceable than a new one, and therefore should be carefully preserved. It is inadvisable to draw heavier letters by pressing heavily on the pen, and therefore, to obtain the desired thickness of letter pen of the proper size should always be used.



Courtesy of Wood-Regan Instrument Co.

Fig. 26. Lettering Template

Mechanical Lettering Equipment. There are a number of mechanical lettering devices available for those not well versed in the art of lettering, or for those who do not care to spend the required time and effort in practicing to obtain skill in lettering. These lettering instruments are available for a number of different styles and sizes of letters, but are limited by their inflexibility. A large number of templates is necessary if any considerable amount of lettering is to be done. These instruments are quite simple in their operation. A pen (usually of a special type)

is placed against the template at the desired letter and drawn around the outline of the letter. One type of template is illustrated in Fig. 26.

Paper. The paper used in preparing a graph varies with the purpose of the graph, and ranges from an ordinary sheet of bond paper to illustration board.

If bond paper is to be used it should be of a rag stock, hard sized, and of good erasing quality. For large graphs, illustration boards of one or two ply may be used.

Ink. India ink is generally used for preparing graphs, since it leaves a more permanent water-proof drawing. It can be obtained in many colors, including black, various shades of red, blue, green, violet, yellow, orange, etc.

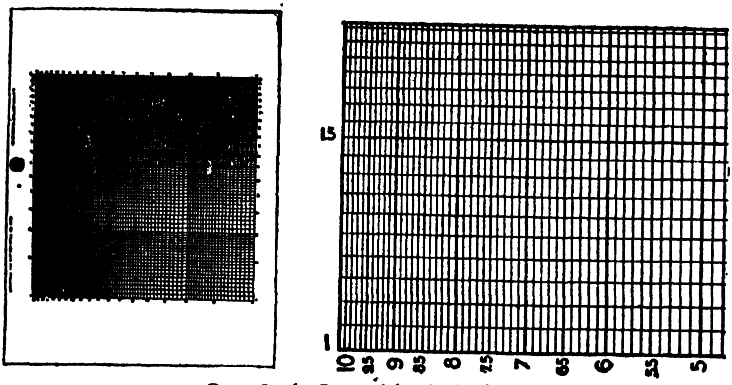
India ink comes in bottles equipped with a quill attached to the cork. The quill is used to fill ruling pens, compass pens, etc. The cork should always be replaced, since the ink evaporates quickly and may thicken and soon become valueless if left exposed.

Printed Graph Sheets. For ordinary work not requiring reproduction, the preparation of the graph is facilitated by using printed background sheets which can be obtained in many forms.[8] The sheets, however, are suitable only for reports, or similar work, since they are not generally available in a large size and do not lend themselves to reproduction.

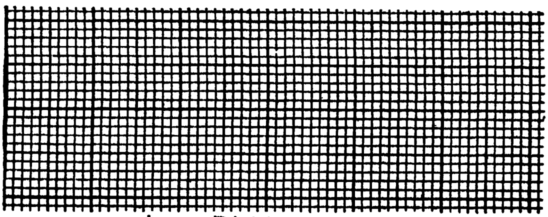
Sheets may be obtained with backgrounds printed in light green, blue, or orange, on either bond or tracing paper. The size of the sheets is generally either 8½x11 or 11x16½ inches, although a few types are prepared in either a larger or smaller size.

These sheets are printed in a variety of rulings, including a number of flexible polypurpose sheets which can be adapted to

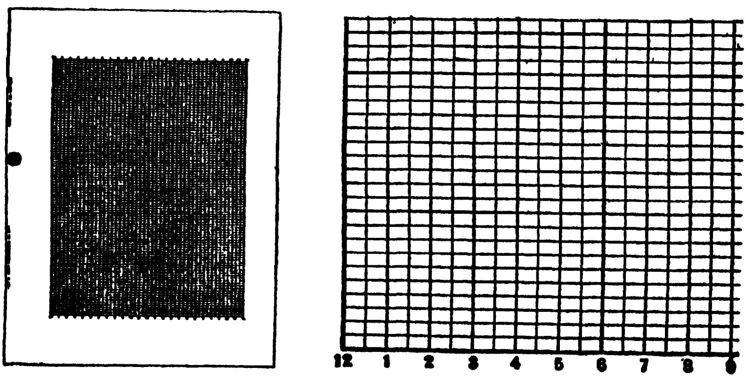
[8] Companies publishing these sheets include the Codex Book Co., Norwood, Mass.; Keuffel and Esser Co., N. Y.; Eugene Dietzgen Co., N. Y.



One Cycle Logarithmic Ruling

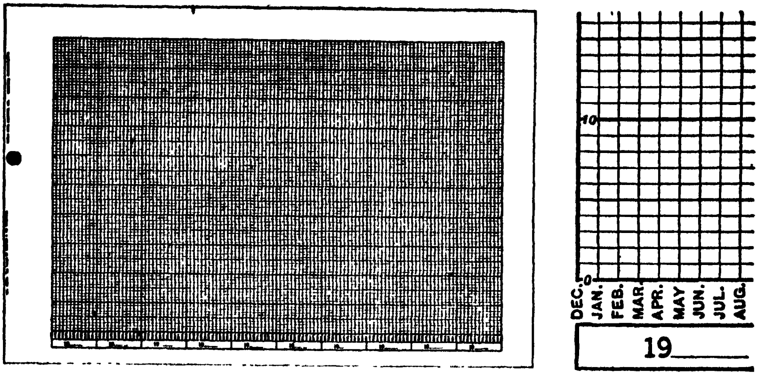


20 by 20 Divisions to the Inch

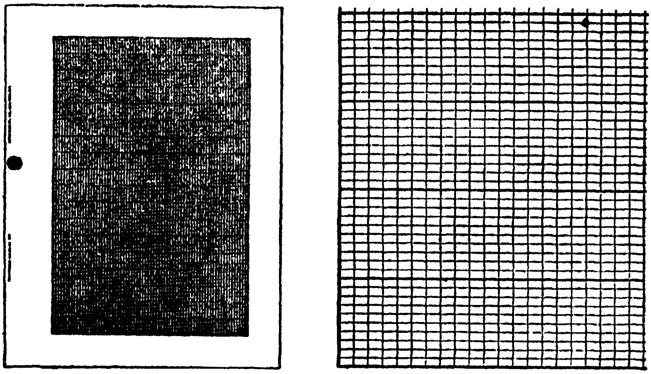


One Day by Hours

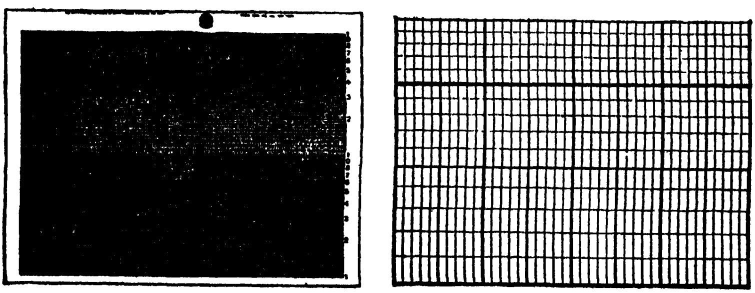
FIG. 27. PRINTED GRAPH SHEETS



Polypurpose



Ten Years by Months



Two Cycle Semi Logarithmic

FIG. 27. CONTINUED

almost any situation. The catalogue of one of these companies lists over 100 different types of rulings.

The variety of rulings available in printed sheets may be seen from a partial list of the types which can be purchased:

A. Various divisions from 1x1 per inch, 2x2 per inch, 3x3 per inch, etc., to 20x20 per inch.
B. More unusual divisions, such as 6x8 divisions per unit, 12x20, 24x100, etc.
C. Rulings for plotting time series, with units of time printed on the axis ranging from one day by hours, one month by days, to fifty years by months.
D. Logarithmic, semi-logarithmic, with from one to five cycles on either axis.
E. Probability, polar coordinate, trilinear rulings, isometric rulings, etc.
F. Polypurpose sheets of various types which can be adapted to many uses.

A few of these sheets are illustrated in Fig. 27.

Printed sheets are prepared in standard sizes and generally have a margin surrounding the grid. A graph will present a much better appearance if only part of the grid is used and the part used cut out and pasted[4] on an ordinary sheet of bond paper. The title, source, and other material may then be typewritten on the surrounding sheet. The much wider margin enhances considerably the appearance of the graph. Fig. 28, which shows the volumetric efficiency of typical oil-sealed and reciprocating pumps, illustrates the use of printed graph paper in graphic presentation.

[4] Rubber cement is recommended for adhesive work in preparing and mounting graphs. It permits the lifting of cemented sheets without tearing, as might happen if ordinary paste or glue were used.

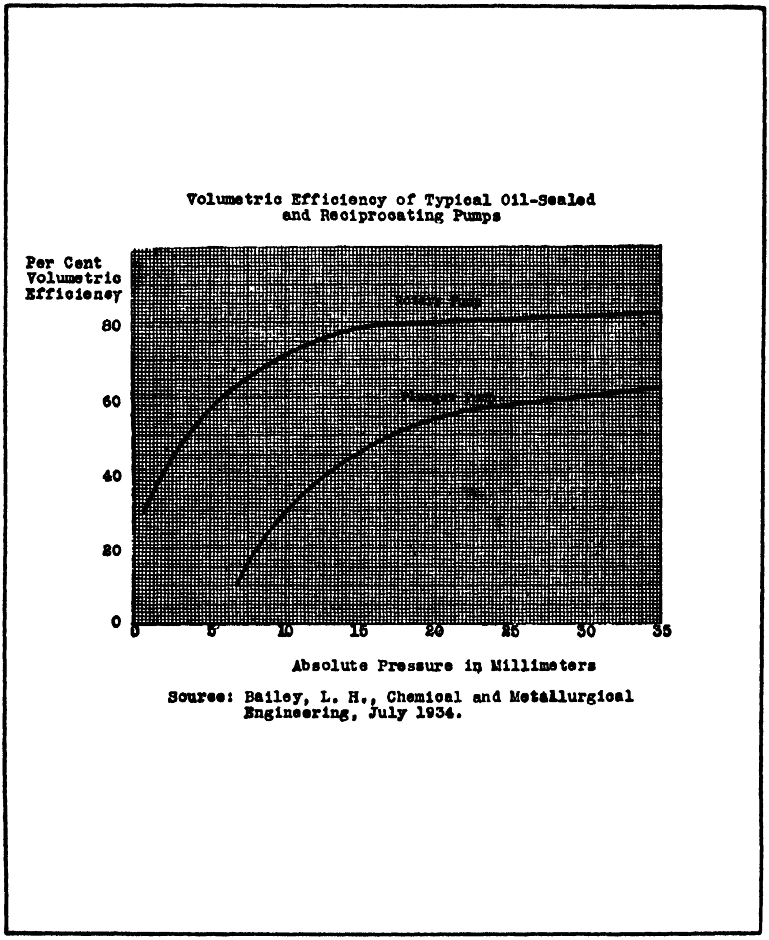


FIG. 28. GRAPH ON PRINTED BACKGROUND

CHAPTER V

THE LINE GRAPH

Types of Line Graphs—When to Use the Line Graph—How to Construct a Line Graph—Scale Caption—Key or Legend—The Title—The Source—Footnotes—Special Types of Line Graphs—The Silhouette Chart—The High-low Chart—The Band Chart—The Shaded or Cross-hatched Graph—The Shaded Line Graph—The Histogram—The Step Diagram—Cumulative Frequency Distribution—Lorenz Curves.

THE line graph, probably the most widely used type, presents varying quantities in the form of a line or curve which moves across the face of a grid or background ruling.

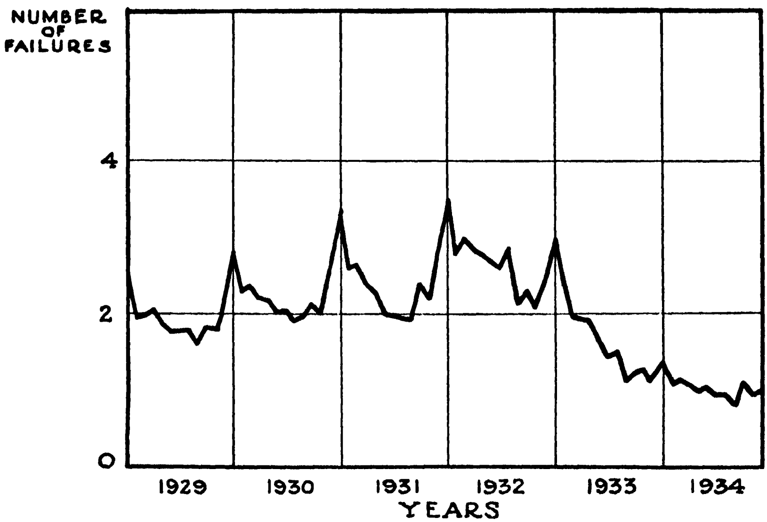
Fluctuations in the line make it possible to obtain a quick visual picture of the "trend" of the data and the individual variations over a period of time or in the size of a variable. A simple line graph showing the number of commercial failures in the United States, 1929-1934, is illustrated in Fig. 29.

The fluctuations within the line show changes from one figure to another. The distance of the line or curve from the base of the graph indicates the size of the quantities that have been plotted.

Types of Line Graphs. Line graphs may be classified in a number of different ways, such as according to the type of data plotted. In order to present data graphically they must first be arranged in some systematic order. Such an arrangement is known as a distribution or series. The various types of groupings, and the names assigned to them, are shown below:

When Data Are Grouped According to	The Resulting Series Is Called a
1. Time of occurrence	1. Time series
2. Magnitude	2. Frequency distribution
3. Geographic location	3. Spatial distribution

In addition, data may be arranged by *kind* or *degree.* Time series and frequency distributions are quantitative arrangements while spatial distributions and distributions by kind or degree are qualitative in nature.



Source: Dun and Bradstreet, Inc.

FIG. 29. COMMERCIAL FAILURES IN THE UNITED STATES, 1929-1934 (LINE GRAPH)

Data grouped according to time of occurrence or by magnitude (the time series and frequency distribution) are generally presented in the form of a *line graph.* This is especially true when there are numerous items in the series.[1]

The *spatial distribution* is generally presented in the form of the *statistical map* (see Chapter X), while the distribution by *kind*

[1] These series may also be presented in other types of graphs. However, the line graph is the most effective form, unless the items are few in number.

or *degree* is usually presented in the form of *bar charts* (see Chapter VII), *area diagrams* (see Chapter VIII), or *solid diagrams* (see Chapter VIII).

Diagrams of relationship, picturing the relation between two series of data, are also presented in the form of a line graph (see Chapter IX).

The types of line graphs according to this classification are:

1. The time series.
2. The frequency distribution.
3. The diagram of relationship.

Line graphs may also be classified according to the type of grid used. Various types of background rulings may be used in presenting a series. Therefore this classification may be considered as a subdivision of the classification outlined on page 50. The grids used may have (1) a scale in which constant absolute quantities are represented by constant distances, (2) scales in which constant percentage changes are represented by constant distances, and (3) other scales which do not use fixed distances to represent either fixed absolute quantities or constant percentage changes.

When to Use the Line Graph. The line graph is used in preference to other types of graphs when there are a considerable number of values to be plotted. It is also to be used where the data are *continuous*, that is where there are no breaks in the series of values presented.[2] The line graph is the most effective type of graph to present the variations in a quantity at many different periods of time, or for the presentation of data, grouped according to size, where there are a considerable number of groups.

How to Construct a Line Graph.

1. Prepare a *grid* or *background* ruling.[3] Since in this ex-

[2] In the discrete, or non-continuous, series only limited gradations are possible. In a series constructed by counting the number of students in classes, for instance, no fractional values can be found.

[3] The method of grid construction is discussed on page 11.

ample an arithmetic grid is to be used, the background rulings (coordinate lines) are uniformly spaced (see Fig. 30). A minimum number of background lines should be used, since when many background rulings are made it is difficult to read the curve line on the graph. As an aid in plotting, numerous lines may be drawn lightly in pencil and later erased.[4] If few background lines are used, a graph will be

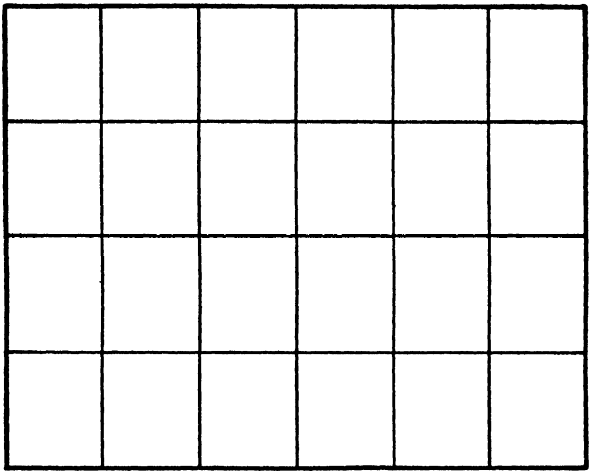


FIG. 30. AN ARITHMETIC GRID

more effective in its presentation and present a much better appearance. An example of a graph with too many coordinate lines, as compared with one properly constructed, is shown in Fig. 5, p. 13. The greater effectiveness of the grid with the fewer background lines can be clearly seen.

2. Select a suitable scale of values and arrange them along the horizontal and vertical axes of the grid. The principles of the preparation of a scale are given on pages 12 and 13.

The values indicated at the intervals on the scale (points of intersection of the background lines and the axes) should be in

[4] Specially prepared printed sheets may be used for plotting. On this type of paper numerous background rulings are printed in light green or orange color, and therefore do not confuse the plottings (see p. 44).

round numbers (such as 2, 4, 10, 20, 25, 50, 100, etc.) rather than in odd or unusual numbers. This will facilitate the location of the points to be plotted.

The horizontal scale (X axis) is used to indicate the periods of time in presenting a time series. The space between any two of the coordinates may be used to represent the period of time (day, month, year, etc.), or the coordinate itself may be used to locate the time interval.

3. The points are then located on the grid by reference to the scales on the X and Y axes. The method of locating the points for plotting is discussed in detail on page 5. The plotted points are joined by a series of *straight* lines. The line drawn by this method should *not* be smoothed into a regular curve unless a mathematical function is being presented or unless the curve has been smoothed by a mathematical procedure.

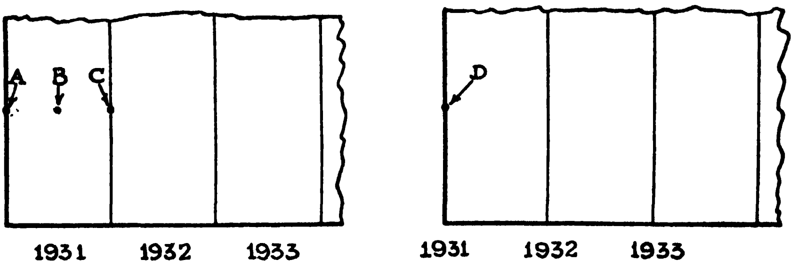


FIG. 31. PLOTTING A TIME SERIES

A value for a given period of time, as for a year, is plotted at the center of the space allotted (at point *b*, Fig. 31). However, if it is specifically indicated that the data represent the beginning or end of the period, it is plotted at points *a* or *c*, respectively. If the coordinate lines are used to represent the time interval, the value is plotted on the coordinate line itself (point *d*).

When a frequency distribution is graphed, the coordinate lines are used to indicate the group limits. In Table II the first group

(class interval) contains values ranging from 50 to 69.9. The second group ranges from 70 to 89.9. The first interval on the horizontal (X axis) is used to indicate the first class interval. The lower limit of the group, 50, is placed at the first coordinate in the section, while the upper limit, 69.9, is placed at the second. But this upper limit for all practical purposes coincides with the lower limit of the next group (or 70), and therefore may be used in its place.[5] A differentiation is made in the frequency distribution itself between the upper limit of one group and the lower limit of the next (69.9 and 70) merely to allocate the borderline cases (such as 70). In practice, then, only the lower limit values of the various groups are indicated, since the lower limit of one group is used as the upper limit of the previous group. The

TABLE II

DISTRIBUTION OF INTELLIGENCE QUOTIENTS IN A LOS ANGELES JUNIOR HIGH SCHOOL IN 1933

Intelligence Quotients	Number of Students
50– 69.9	64
70– 89.9	581
90–109 9	1,304
110–129.9	714
130–149.9	124
150–169 9	13
	2,700

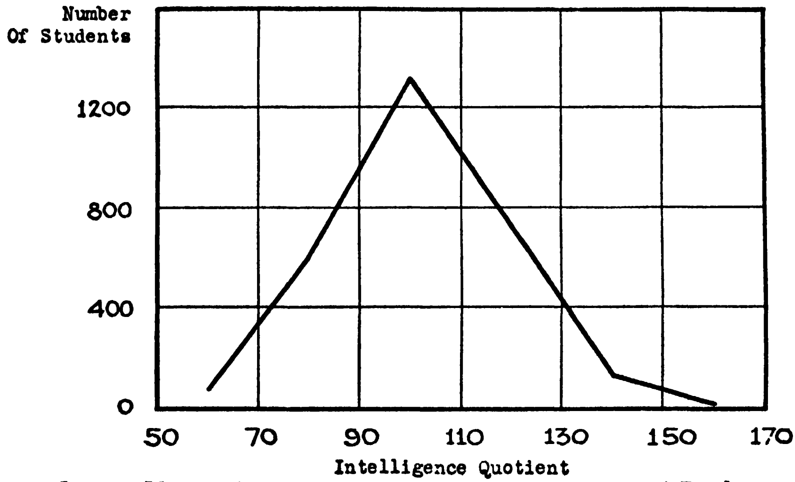
Source: Humm, D. G. and Humm, K. A., "Coefficient of Rank," *Journal of Educational Psychology*, May, 1933.

vertical axis (Y axis) is used for a scale to indicate the number of cases falling within each group. In Fig. 32 below there is presented graphically the distribution of intelligence quotients in a Los Angeles junior high school in 1933, in which the Y axis

[5] Since the group includes all values ranging between 50 and under 70, the upper limit should read 69.9999, etc., or for practical purposes 70.

indicates the number of students and the X axis, the intelligence quotients.

There is no indication in the specified number for each class interval to show the distribution of the cases between the upper and lower limits of the group. Therefore it is assumed that the cases are distributed evenly through the class interval, and that the mid point, a value halfway between the upper and lower limit of the group, represents the average value of the class interval.



Source: Humm, D. G. and Humm, K. A., Coefficient of Rank, Journal of Education Psychology, May, 1933

FIG. 32. DISTRIBUTION OF INTELLIGENCE QUOTIENTS IN A LOS ANGELES JUNIOR HIGH SCHOOL IN 1933 (LINE GRAPH)

To indicate graphically the number of cases falling within the group, the value is plotted at a point halfway between the two coordinates for the group (point e), at the place indicated by the appropriate value on the vertical (Y) axis scale.

Scale captions indicating the units used are placed on the horizontal and vertical axes. The scale caption for the horizontal axis (X axis) is placed at the center directly beneath the scale, and that for Y axis at the top of the left vertical axis (as shown in Fig. 32).

The lettering on the scale captions should be parallel to the base of the graph, so that they can be read without turning the page. This rule should be followed in all lettering on the graph.[6]

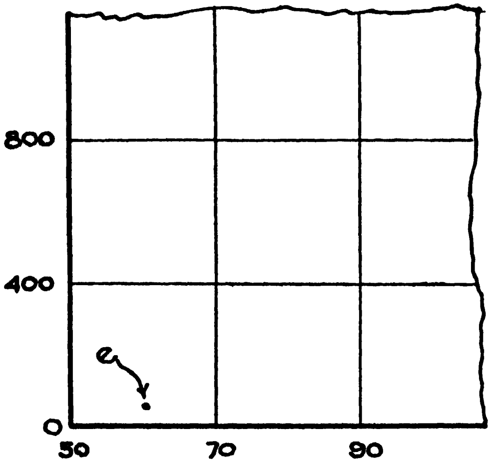


FIG. 33. PLOTTING A FREQUENCY DISTRIBUTION

The Curve Line. The line on the graph presenting the variations in the data should be drawn more heavily than the background lines in order that it may stand out sharply against the grid.

If there are several variables, and hence several curve lines on the same grid, different types of lines should be used for each of the curves in order to distinguish each curve. This is especially important if any of the curve lines should cross or come into close proximity. In order to distinguish curves, lines of the types indicated in a, b, c, or d in Fig. 34, or any variations upon them, may be used.

Lines of the types shown in e, f, or g should be avoided, since the markings placed on the lines to distinguish them from one another may be misinterpreted as the location of the plotted points.

[6] On pages 11-18 there is discussed in detail the procedure in constructing scales and the general rules to be followed in lettering, etc.

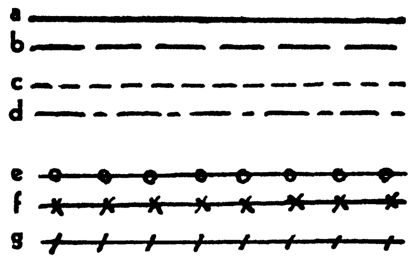
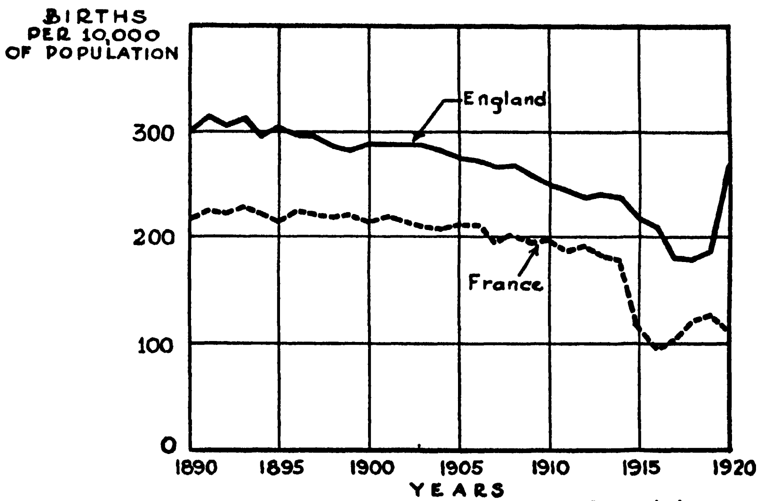


Fig. 34. Types of Curve Lines

Key or Legend. If several lines are drawn on the same graph they must be identified. For this purpose a key may be used or the lines may be labeled. The key or legend should be enclosed in a



Source: Pearl, Raymond, Medical Biometry and Statistics, W. B. Saunders Co., 1930, pp. 224-225, Table 20

Fig. 35. Crude Birth Rates in England and France, 1890-1920 (Illustrating Use of Arrows)

box and placed in some clear space on the face of the graph. The key should be placed some distance from the curved lines, or borders of the graphs. A short line of the type used for each of the curve lines is placed within the box and identified.

If it is not desired to use a key, labels may be used to identify

several lines. The labels are placed next to the curved lines at some convenient position. Arrows may be used to link the line and label. Labels should not be lettered along the curved lines, but, like all other lettering on the graph, should be parallel to the base of the graph. The proper use of arrows on a graph may be seen in Fig. 35, showing crude birth rates in England, 1890-

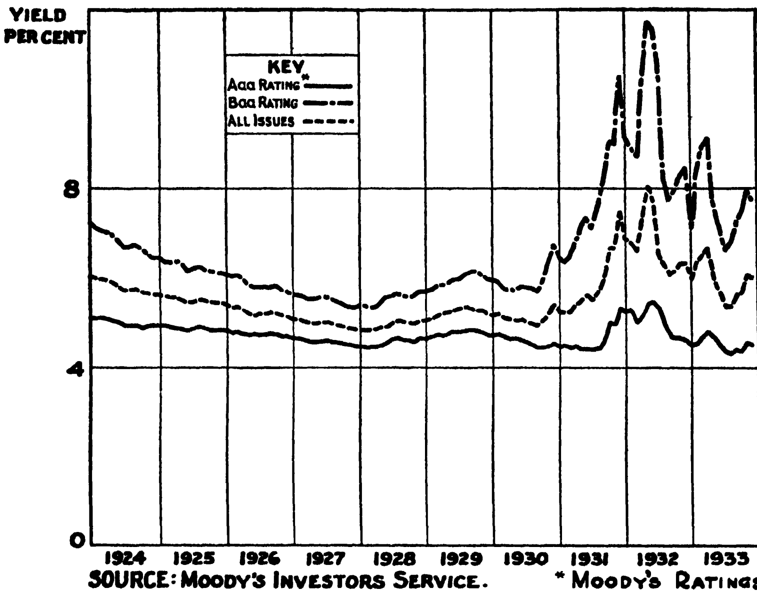


FIG. 36. HIGH GRADE AND LOW GRADE BOND YIELDS, 1924-1933 (ILLUSTRATING THE BOXED-IN KEY)

1920, while the use of a key is shown in Fig. 36, presenting high-grade and low-grade bond yields, 1924-1933.

The Title. The subject of the graph should be completely defined and explained by the title. It should not be necessary to resort to the text for an explanation. The title should specify the nature of the data, the territory covered, and the period of time dealt with. These elements of the title customarily appear in the order named.

Generally the title is placed above the graph, in lettering somewhat larger than the other lettering. When the graph is presented on a printed page, the title is placed beneath it. For a more complete discussion of the title, see page 16.

The Source. The source of data from which the graph was constructed is always indicated on it, except where original data have been obtained. On a line graph, the source is placed below, at the lower left-hand side. The source provides the reader with the authority for the data, is a means for checking the data, and is also used as a reference for additional information.

Footnotes. Footnotes are used occasionally on graphs to denote the special nature of some of the data, or to indicate special material that may have been included or excluded. On a line graph, footnotes are placed immediately beneath the grid, generally to the right-hand side (see Fig. 37).

Special Types of Line Graphs.[7] Variations of the line graph, such as the silhouette chart, the high-low chart, and the band diagram, are especially useful in presenting certain types of data.

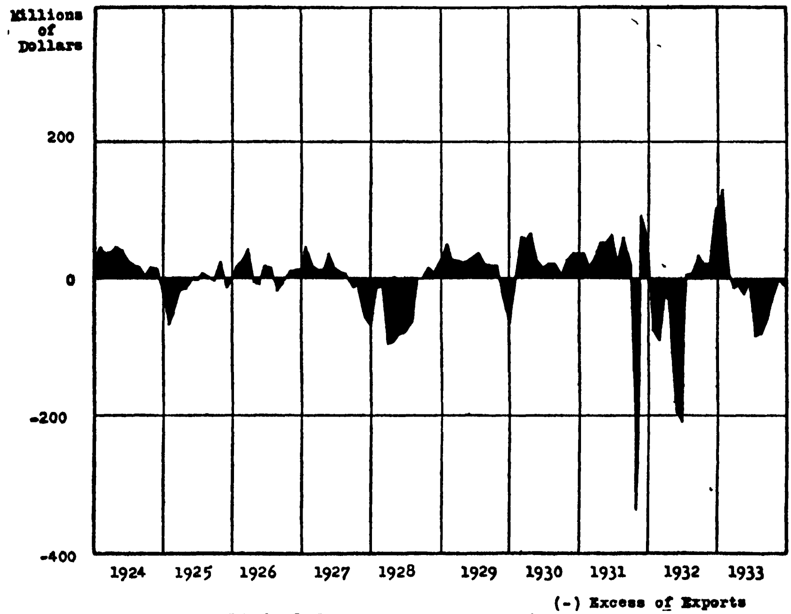
Pictorial Line Graphs. When any considerable number of graphs are included in a single report or publication the presentation tends to become monotonous and loses its effectiveness especially when any single type of graph is used throughout. This is particularly true of a series of line graphs. Variety may be introduced by varying the kind of graph used or, more especially, interest and attention attraction value may be enhanced by introducing the pictorial form of graph.

The pictorial element may be included in the line graph by the use of a picture as a background for the graph, by superimposing pictures or cartoons on the graph, or by using a series of pictures to replace the curve itself. The use of the pictorial background is shown in Fig. 38a while the superimposed picture used to emphasize the subject matter of the graph is illustrated by Fig. 38b.

[7] The line graphs with special non-arithmetic scales are dealt with in detail in Chapter VII.

The Silhouette Chart. The silhouette chart, a variation of the line graph, is one of the most striking types of graphs and has a high attention-attraction value.

Fluctuations from "normal," variations about 100 per cent, plus and minus departures, and other similar variations about a given base line, are effectively presented in this manner.



Source: United States Department of Commerce

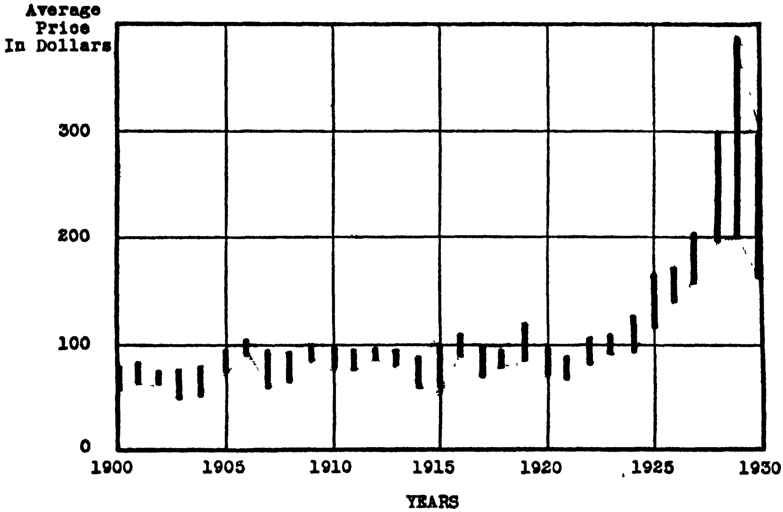
FIG. 37. GOLD MOVEMENTS FROM THE UNITED STATES, 1924-1933 (SILHOUETTE GRAPH)

The construction of the silhouette chart is in every way identical with that of the simple line graph, with the exception that the base line (zero, 100 per cent, etc.) is ruled heavily and the area between the curve line and the base line shaded in. The shaded area presents a sharp contrast to the background, and thus is very effective.

A graph of gold movements in this form is shown in Fig. 37. The base line in this instance is zero, with variations above the line representing imports, and below the line exports. The scale

above the zero line is of positive values, below the zero line negative.

The High-Low Chart. In presenting certain types of data it is frequently desirable to show the variations occurring *within* the specified periods of time (day, week, etc.), as well as *between* the periods of time (day to day, week to week, etc.). This can be

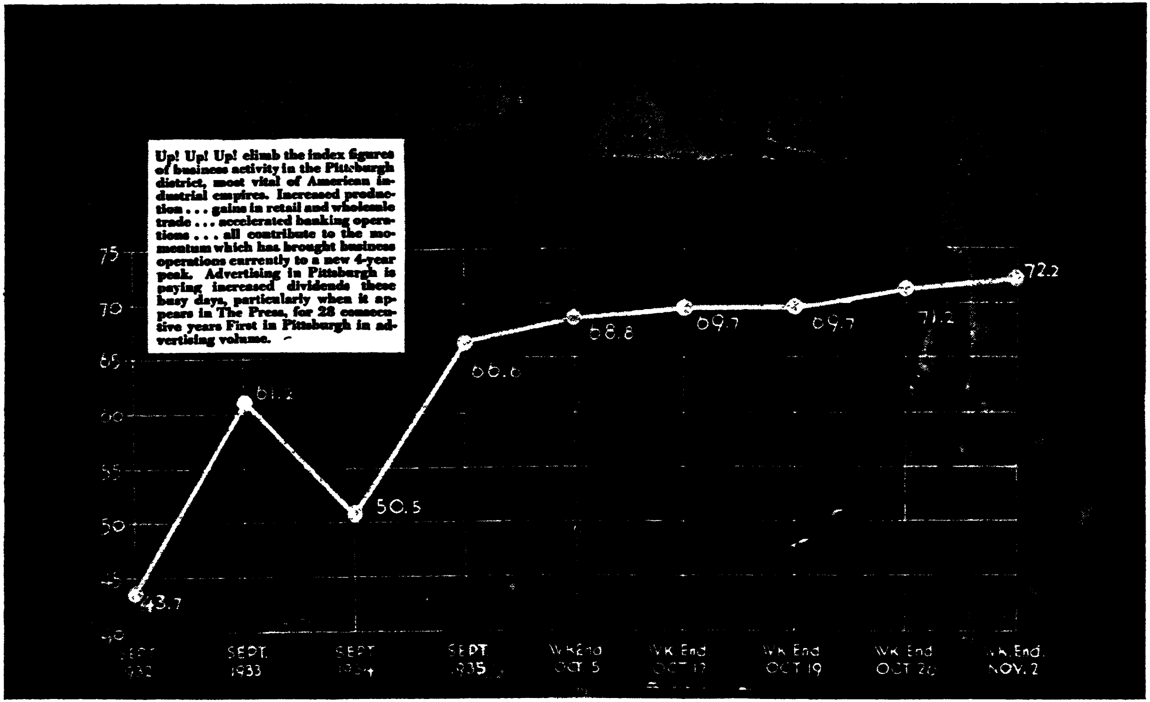


Source: Dow-Jones & Company, 1900-1930

FIG. 38. STOCK PRICE AVERAGES (HIGH LOW GRAPH)

accomplished by indicating the high and low values for each period of time. An especially effective use of the high-low values is in the presentation of price data.

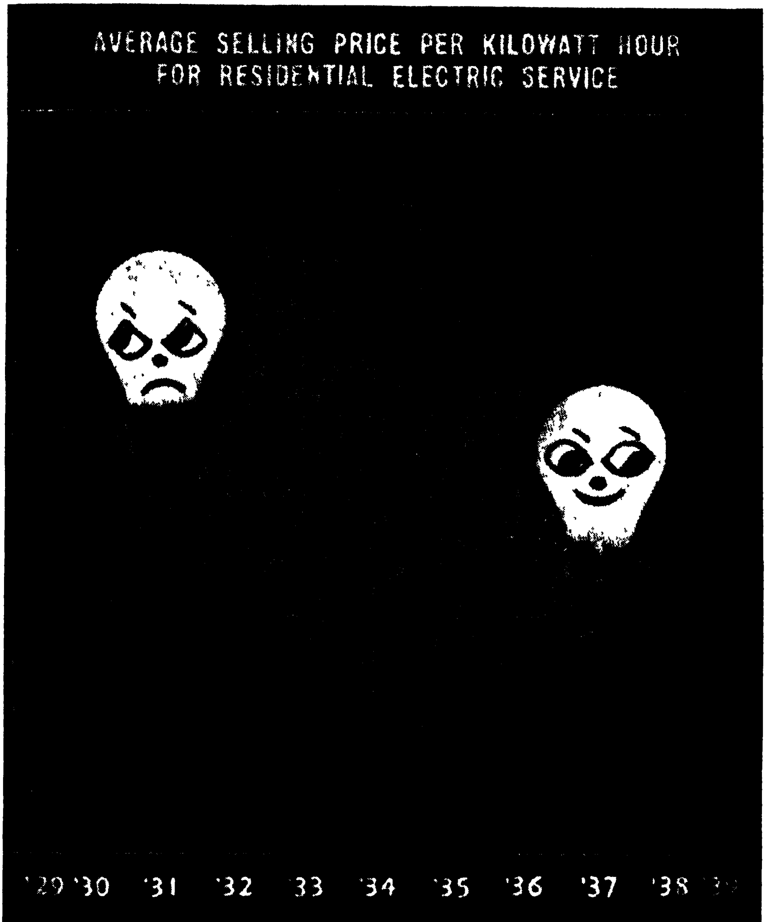
A table of the "highs" and "lows" by days, weeks, months, or other periods is obtained. Then the high and low points for each period are located on the graph. If a line is drawn between the "high" and the "low" for the same period, a series of lines will result which, when closely spaced, give rise to the appearance of a band which moves across the face of the graph. The band will show a period-to-period change, and also variations within the period as well. Thus, in Fig. 38, the continuous movement of prices during 1900-1930 may be seen, as well as the range of prices as of any particular year.



Index Numbers reported by Bureau of Business Research University of Pittsburgh. Photograph by Luke Swank

Reproduced from "Printers' Ink," November 21, 1935, p. 61

FIG. 38a. LINE GRAPH WITH PICTORIAL BACKGROUND



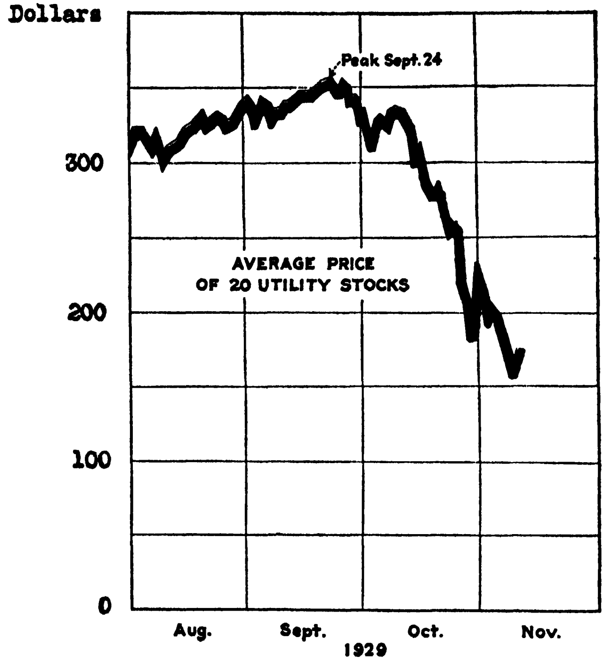
Reproduced from "Fortune," May, 1940, p.164

FIG. 38b. A PICTORIAL LINE GRAPH

The "closing" prices, average value, or other desired figures, may be indicated by plotting that value, and joining the points with a series of straight lines. This type of graph then becomes a combination of the ordinary line graph and the high-low chart.

Another method is to join all the "highs" with a series of straight lines and all the "lows" in a similar fashion, and then shade the area between. An example of this type of high-low graph is shown below in a chart illustrating security fluctuations.

In this case the high point and the low point of the average of twenty utilities were plotted daily over a period of four months in 1929. The plotted high points were connected by straight lines to form a curve. The low points were connected in a similar manner. The area between the two curves was then shaded.



Reproduced from "Electrical World," January 4, 1930
FIG. 39. A HIGH LOW CHART

The Band Chart. The band chart, another form of line diagram, is used to show the variations occurring in the component parts of total figures, as well as the fluctuations in the total figures themselves.

The preliminary steps in the construction of the band chart (the preparation of the grid, scales, etc.) are the same as those for the simple line graph. The plotting of the graph, however, is somewhat different.

The first step in constructing a band chart is to so arrange the data that the components or segments are placed in order of size (the largest first, etc.).

The values for the largest segment (in this case manufactured goods for a graph of Exports by Type of Merchandise) are then plotted in the usual fashion. The area under the resulting curve is shaded or cross-hatched.[8] The figures for the next component part (Crude Material Exports) are then added to those already plotted, and the resulting data are plotted. The area between this new curve and the previously shaded area may now be cross-hatched with a different type of cross-hatching.

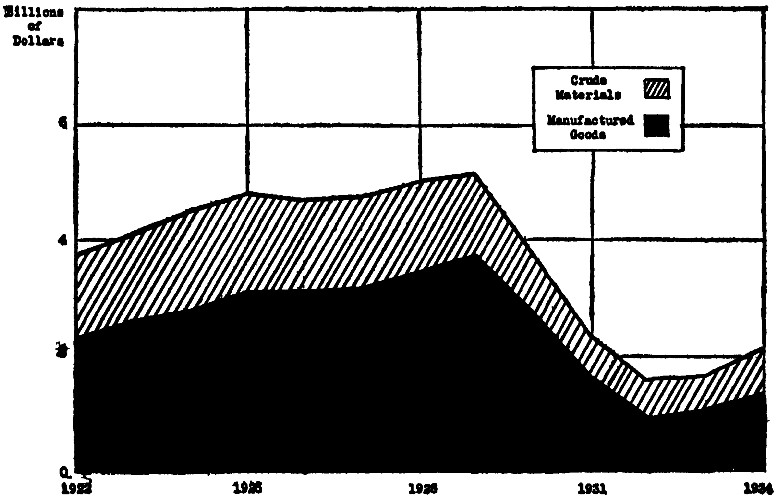
This process is continued until all the component parts have been included. The plotted points of the final segment will be equal to the total value, since they are computed by cumulating *all* of the component parts. Thus, for 1922, the plotted point of the last segment was 3832 millions of dollars, or a sum equal to all of the segments.

The variations of the top line in the band chart—*i.e.,* the changes in its distance from the horizontal (X) axis, indicates the fluctuations in the total which represents the sum of all the components. However, if the interest is confined to fluctuations in the value of any one component, the base for the figures will be a varying rather than straight line (unless it is the bottom component) and the variations in the *width* of the appropriate segment on the chart must be examined. For example, the band representing crude material exports in Fig. 40 was determined by adding the figures for that component to that of the previous component. Therefore it will be the distance of the top line of this segment from that of the previous segment (or the width of the segment considered) that will determine the size of the value for that component at any given point.

Considerable difficulty may be found in comparing the width

[8] See page 153 for methods and types of shading and cross-hatching.

of a component band at two different points if a sharp change should occur in the preceding segment between the two points.



Source: United States Department of Commerce

FIG. 40. EXPORTS FROM THE UNITED STATES, BY TYPE, 1922-1934 (BAND CHART)

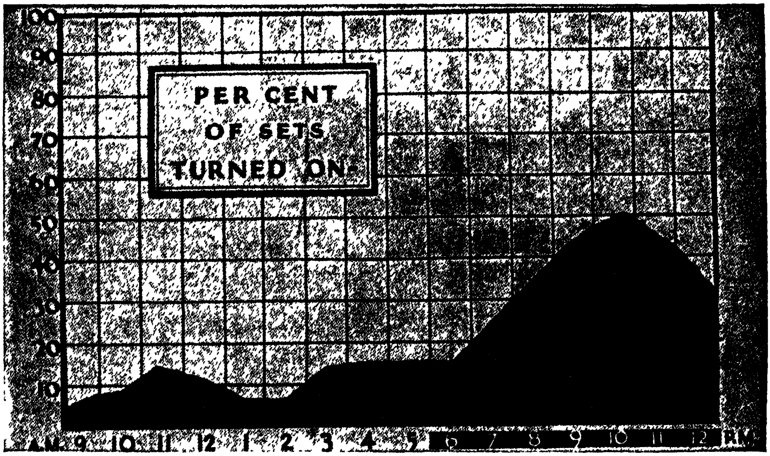
The Shaded or Cross-hatched Graph. The line graph can be made much more effective in its presentation by shading or cross-hatching (filling in with diagonal rulings) the area under the curve. The effect obtained by this technique makes the graph more unusual in its appearance and therefore of much greater attention value.

Fig. 41 illustrates a curve of this type. The curve plotted represents the per cent of total radio sets in operation during the various hours of the day and night as determined from a survey conducted by the American Newspaper Publishers' Association. The graph in this form is essentially a single component band chart.

The shaded type of graph is especially effective in this instance, since the height of the grid represents 100 per cent and the width

of the band on the chart represents the proportion of the total (100 per cent) of the sets in operation.

In certain types of data the area *between* two curves plotted on the same grid is of particular significance. For instance, if the income and costs of a business organization are graphed on a single grid (of course using the same scale), the area between the lines will represent the profit or losses, depending on which of the two curves is higher. This kind of graph may be used



The largest number of radio sets are in use from 9 to 10 P.M.

Source: American Newspaper Publishers Association, The Newspapers 1934

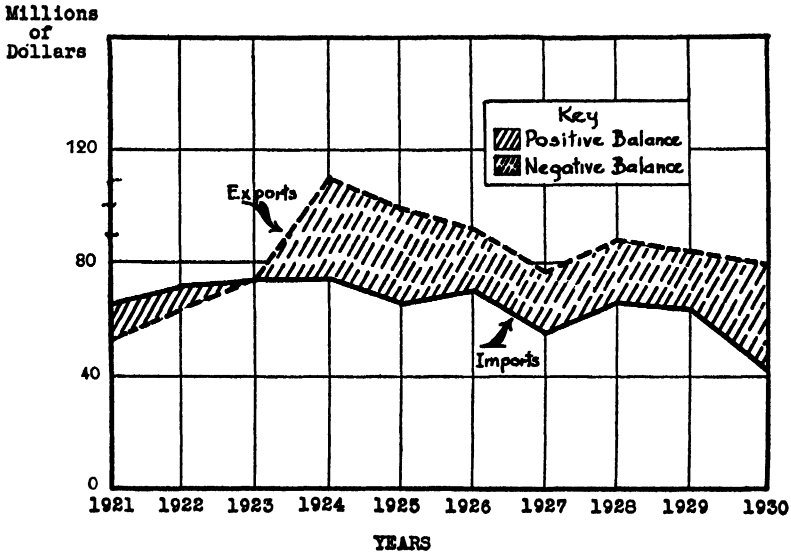
Fig. 41. A Shaded Line Graph

whenever the difference between two curves represents a balance. A shaded graph permits simultaneously a study of not only the variations in profits, but also of income and expenditures.

To prepare a graph of this type, both curves are plotted on the same grid and the area between them shaded or cross-hatched. One type of cross-hatching may be used for a positive balance, and another for a negative balance.

In Fig. 42 the cross-hatched area between the two curves is the balance of trade, since the two curves plotted are exports and imports.

The Histogram. In the section on the preparation of the graph of the frequency distribution it was pointed out that the distribution of the cases between the upper and lower limits of a class interval was unknown. Therefore it was assumed that the cases were distributed evenly throughout the group, and that the mid-point could be used to represent the cases in the class interval.

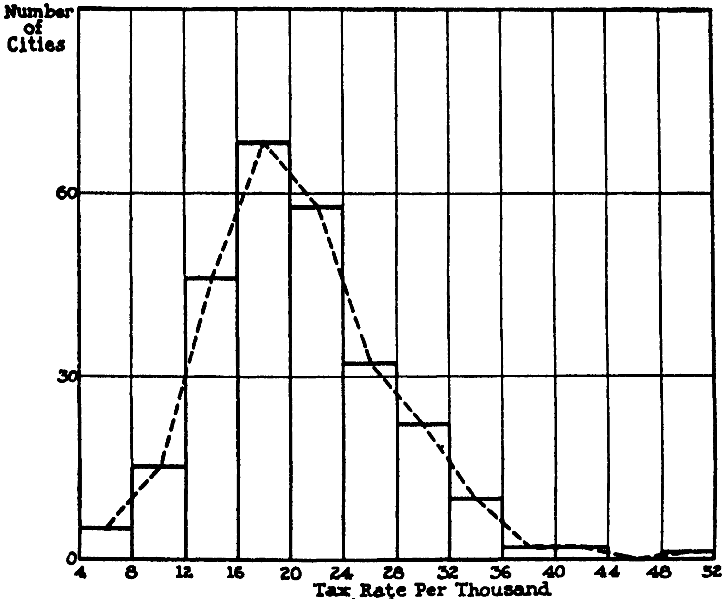


Source: United States Department of Commerce

FIG. 42. SILVER EXPORTS AND IMPORTS OF THE UNITED STATES, 1921-1930 (SHADED LINE GRAPH)

Another method of presenting the same data is to draw a horizontal line across the graph, from the lower limit of the group to the upper limit, at a height selected by reference to the Y scale, indicating the number of cases in the group. If lines vertical to the X axis are then dropped from the ends of the horizontal line, a rectangle is formed. If this procedure is continued for all points, the resulting graph will consist of a series of closely spaced rectangular bars. This type of diagram is known as a histogram, rectangular frequency polygon, or column diagram. In form it is

similar to the bar chart (see Chapter VII), since the lengths of bars, of uniform width, are compared. However, in the histogram there is no spacing between the bars as is the case in the true bar chart.



From Arkin, Herbert and Colton, Raymond R., "Statistical Methods," Second Edition, Barnes and Noble, 1935

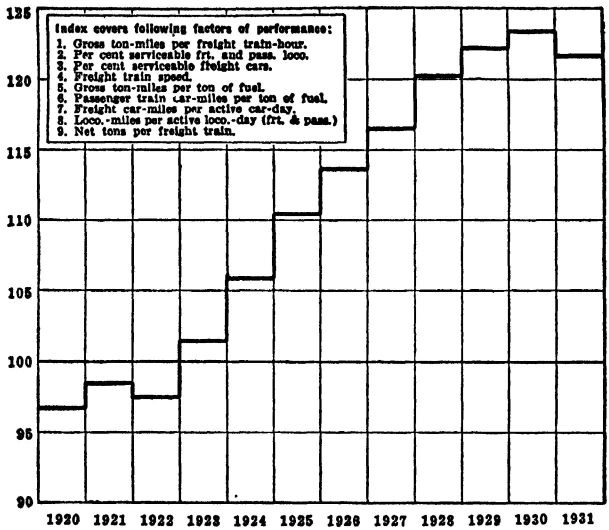
FIG. 43. DISTRIBUTION OF TAX RATES ON "TRUE" VALUATION FOR 261 CITIES IN THE UNITED STATES, 1927

The Step Diagram. When graphing a time series the total for each time interval was plotted at a point halfway between the beginning of the period and the end of the period. As in the case of frequency distribution, this was done because the actual distribution of production throughout the period was unknown, and therefore it was assumed that it was evenly distributed throughout the period. A technique similar to the histogram may be used here by drawing a horizontal line across the time interval at the proper height. In this type of diagram, however, instead

of dropping a line to the X axis, a vertical line is drawn at the beginning and end of each period to meet the horizontal line of the previous or next period. The resulting graph is known as a step diagram. An example of this type of graph may be seen in Fig. 44, which presents the Index of Railway Operating Efficiency, 1920-1930.

Index of Railway Operating Efficiency, 1920 to 1931

Average Five Years 1920-1924 Equals 100



NOTE: Surplus or stored units of equipment are eliminated from computations underlying Factors 7 and 8. Factors 1 and 5 are exclusive of locomotive and tender.

Source: Bureau of Railway Economics

From Moody's Manual of Investments, Railroad Securities, 1932

FIG. 44. STEP DIAGRAM

Cumulative Frequency Distribution. As a further step in the analysis of the frequency distribution, the value in a series may be cumulated. A cumulative frequency distribution is known as an ogive.

The frequency distribution may be cumulated in two ways. A count may be made of the number of cases occurring above each specified value (generally the lower limits of the class interval

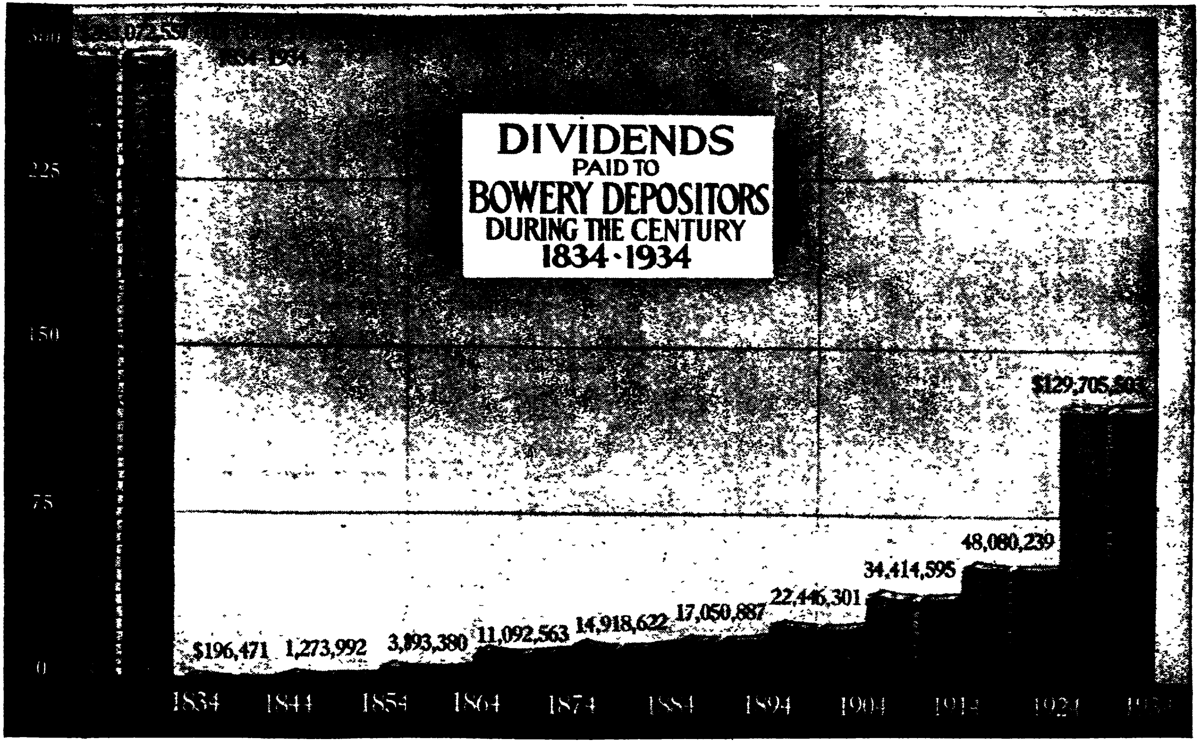


Fig. 44a. A Pictorial Step Diagram

groupings) to result in an "and over" group, or the analysis may be accomplished by determining the number of cases below each given limit to obtain an "and under" distribution.

The curve resulting from plotting the original data in cumulative form is generally smoothed. A frequency distribution is shown cumulated in both ways below:

TABLE III

DEATH RATES IN CITIES OF OVER 100,000 POPULATION IN THE UNITED STATES IN 1930

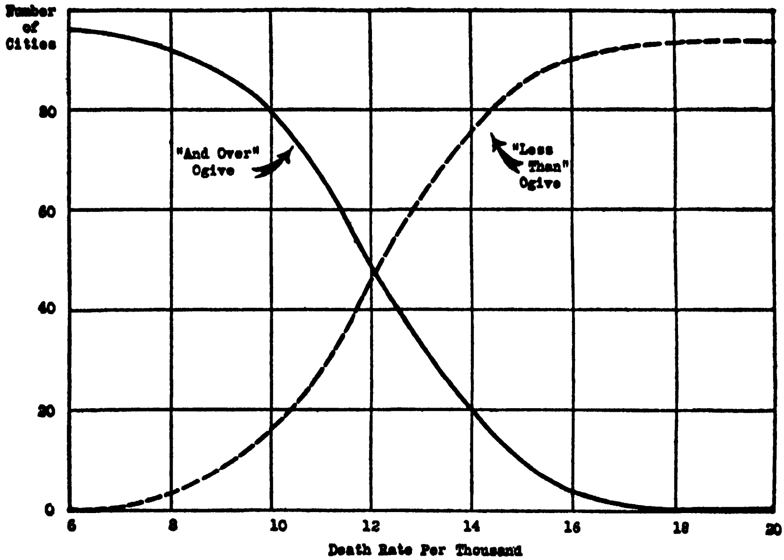
Death Rate per Thousand	Number of Cities	Death Rate per Thousand	Number of Cities
6 and over	94	Less than 6	0
7 and over	93	Less than 7	1
8 and over	92	Less than 8	2
9 and over	87	Less than 9	7
10 and over	79	Less than 10	15
11 and over	67	Less than 11	27
12 and over	46	Less than 12	48
13 and over	32	Less than 13	62
14 and over	21	Less than 14	73
15 and over	8	Less than 15	86
16 and over	4	Less than 16	90
17 and over	1	Less than 17	93
18 and over	1	Less than 18	93
19 and over	1	Less than 19	93
20 and over	0	Less than 20	94

Source: United States Bureau of the Census

A glance at the table will indicate the analytic advantages of an arrangement of data in cumulated form.

When graphed, the ogive runs diagonally across the chart in the form of an "S." The "and over" ogive starts at the upper left-hand side of the chart and descends diagonally across the grid to the lower right-hand corner. The "and under" ogive starts at the lower left-hand corner of the grid and rises diagonally to the upper right-hand corner.

The Lorenz Curve. When it is desired to make a comparison of the number of items in the various groups or class intervals of a frequency distribution, and the total *value* attributable to that group, a special type of graph known as the Lorenz curve is used. The best-known graph of this type is the comparison of



Source: United States Bureau of the Census

FIG. 45. DEATH RATE IN CITIES OF OVER 100,000 POPULATION IN THE UNITED STATES IN 1930 (OGIVES)

the number of people in the various income groups and the total amounts of their incomes.

The first step in the construction of the Lorenz Chart is to convert both the number and the value for each group into per cent of the total number and value, in the form of a "less than" ogive. In Table IV, below, the distribution of wholesale sales by size of firm is shown for the United States for 1930. In the table the conversion to percentage form is also given.

A grid is then prepared with two percentage scales. On the

TABLE IV

DISTRIBUTION OF WHOLESALE SALES BY SIZE OF FIRM FOR THE UNITED STATES, 1930

Size of Firm (Thousands of Dollars of Sales)	Number of Firms	Volume of Sales (in dollars)	Per Cent of Total Number	Per Cent of Total Volume
Less than 25	14,235	$ 158,262	18.6%	.6%
Less than 50	23,568	496,774	30.8	1.8
Less than 100	36,154	1,407,366	47.3	5.1
Less than 200	49,667	3,340,599	64.9	12.2
Less than 300	57,081	5,154,277	74.6	18 9
Less than 400	61,701	6,750,128	80.6	24.7
Less than 500	64,716	8,093,920	84.5	29.6
Less than 1,000	71,453	12,771,003	93 3	46.8
Less than 25,000	76,600	27,231,833	100.0	100.0

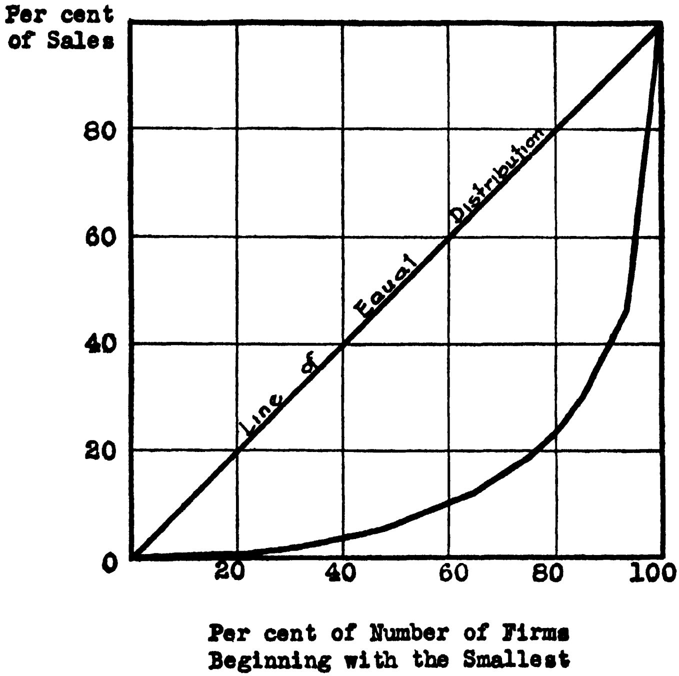
Source: United States Department of Commerce, 1930 Census

X axis the scale is for the number of firms in terms of per cent, and ranges from 0 to 100 per cent. On the Y axis the scale is for the total sales. In this graph the X axis has percentage of total firms beginning with the smallest as the scale for the X axis and percentage of total sales on the Y axis.

If the business was distributed evenly—that is, in direct proportion to the size of the firm—10 per cent of the firms would do 10 per cent of the business, 20 per cent of the firms, 20 per cent of the business, etc. This even distribution may be indicated by a diagonal drawn from the lower left-hand corner of the grid to the upper. This line, known as the *line of equal distribution*, serves as a basis of comparison.

The data for the graph are plotted in the following manner. Starting with a point of 0 per cent on the X axis and 0 per cent on the Y axis, the next point would be located at 18.6 per cent on the X axis (for the per cent of smallest firms) and .6 per cent on the Y axis (representing the per cent of the total amount of business for these firms). The next point would be 30.8 per cent on the X axis and 1.8 per cent on the Y axis. This is continued until

the last point is plotted at 100 per cent on the X axis and 100 per cent on the Y axis. The completed chart is shown in Fig. 46.



Source: United States Department of Commerce 1930 Census

FIG. 46. DISTRIBUTION OF WHOLESALE SALES BY SIZE OF FIRM FOR THE UNITED STATES, 1930 (LORENZ CURVE)

The greater the departure from the line of equal distribution the greater the disparity of the distribution.

A great many different series lend themselves to this type of analysis.

CHAPTER VI

THE RATIO CHART

The Principle of Logarithmic Rulings—Types of Logarithmic Rulings—Characteristics of Logarithmic Charts—Uses of Logarithmic Charts—How to Construct a Logarithmic Chart—Construction of a Logarithmic Scale—Construction of a Flexible Scale—Per Cent Scales—Frequency Distributions on Logarithmic Paper—Other Rulings—Polar Coordinate Paper—Trilinear Diagrams—Other Special Rulings—Probability Ruling—Rules to Avoid Common Errors in Ratio Charts.

The graphs dealt with thus far have had grids which were arranged so that equal spaces on the background ruling always indicated equal quantities, since the coordinate rulings were spaced equally.

To study the relative or percentage fluctuations, rather than the actual changes in data, a ruling is used in which the coordinate lines are not equally spaced. This type of ruling is called a logarithmic grid.

If a graphic comparison is to be made of the sales of a business concern for a small territory as compared with a larger territory such as the entire United States, when plotted on an arithmetic grid the resulting graph is likely to be misleading. Although the sales for both the small territory and the entire United States may have shown the same relative increase (for example, 10 per cent), there will be a great difference in the absolute amounts. For the small territory an increase of 10 per cent may be the result of a rise of only $10,000 in sales, for the entire United States an increase of $100,000 might be required for an equal percentage increase.

Since the scale on arithmetic grids is in terms of absolute amounts (dollars in this instance), the curve for the United States would rise ten times as much as that for the small territory, although the increase is *relatively* the same. However, if a logarithmic ruling is used, no matter what point they are measured from, the two curves will rise the same distance since equal distances on the logarithmic scale always represent equal percentage changes.

The Principle of Logarithmic Rulings. When there is a constant percentage change between two pairs of figures, the differences between the logarithms of the figures will be equal.[1]

Thus if percentage changes are to be studied, the original data may be converted into logarithmic form and plotted as usual. If this is done, constant percentage differences or changes will be represented by equal distances on the scale. For instance, if there is an increase of 50 per cent in both of two sets of figures, both curves will rise an equal distance on the face of the graph.

However, since a great deal of time and effort is required to convert the original data to logarithms,[2] and since it is difficult to locate the rather extended figures of logarithms on the scale, a more convenient procedure is to arrange the scale so that the numbers may be plotted directly in terms of their logarithms.

If such a scale is not used, the original data must first be converted to logarithmic form and then plotted in the usual fashion. Thus, if the value 5 is to be plotted, its logarithm is first deter-

[1] A logarithm is the power to which 10 must be raised to obtain a given number—*i.e.*, $10^0 = 1$, $10^1 = 10$, therefore the logarithm of 1 is 0, and the logarithm of 10 is 1.

Number	*Logarithm*	
2	0 30103	
4	0.60206	
Difference	0.30103	100% increase
5	0.69897	
10	1.00000	
Difference	0.30103	100% increase

[2] The conversion may be carried out by reference to a table of logarithms.

mined (0.69897) and then plotted. It is possible, however, to prepare a scale in advance so that the point corresponding to the position 0.69897 will be marked 5. The data may then be plotted without resort to the intermediate step of determining the logarithms of the original figures.

A simple arithmetic scale may be readily converted into a logarithmic scale by determining the relation between the natural numbers and their logarithms. Thus:

0.0000 is the logarithm of 1
1.0000 is the logarithm of 10
2.0000 is the logarithm of 100
3.0000 is the logarithm of 1000

The intermediate positions between these given figures may be determined in the same fashion. The relation between a simple arithmetic scale and the same scale for plotting logarithmic values is shown below:

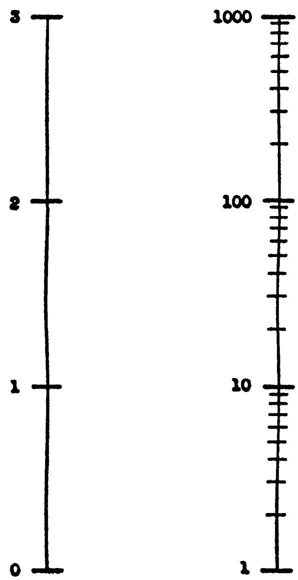


FIG. 47. ARITHMETIC AND LOGARITHMIC SCALES

In order to graph the value 10 in logarithmic form, that value is located on the new scale directly and plotted at that point. The number itself is not converted to logarithmic form. However, plotting the value 10 at this point is equivalent to plotting the logarithm of 10 rather than the actual value 10.

The actual construction of a logarithmic scale may be carried out with the aid of a slide rule which consists of a series of such scales. Printed sheets with logarithmic rulings are readily obtained.

Types of Logarithmic Rulings. If a logarithmic ruling is used on both the horizontal and the vertical axes, the grid is known as a logarithmic grid. If a logarithmic ruling is used on only one axis, with an arithmetic scale on the other axis, the paper is known as a *semi*-logarithmic ruling.[3]

Since time is always placed on the X axis, an arithmetic scale is frequently used on this axis, while the logarithmic ruling is retained on the Y axis.

Characteristics of Logarithmic Charts.

1. To plot zero on the logarithmic scale would necessitate the location of the logarithm of that value on the equivalent arithmetic scale, but the logarithm of zero is minus infinity and therefore cannot be located on the scale.
2. Semi-logarithmic graphs have an arithmetic ruling on one of the scales, generally the X axis, as time is used as the independent variable. Since changes in time increase or diminish by a constant difference, as a year or month, etc., an arithmetic ruling is used to depict these constant changes.
3. When plotted on logarithmic paper, a geometric progres-

[3] Semi-logarithmic paper is also known as arith-log paper.

sion[4] plots as a straight line since the logarithms of a geometric progression form an arithmetic progression.

Geometric Progression	*Logarithms*
2	0.30103
4	0.60206
8	0.90309
16	1.20412

4. Equal rises or falls on a logarithmic graph indicate equal percentage changes in the actual data.
5. Equal slopes on a logarithmic chart denote equal rates of change in the data.
6. An ascending (or descending) curve which approximately describes a straight line in its rise (or decline) indicates a uniform rate of growth (or decline).

Uses of Logarithmic Charts.

1. To study relative or percentage changes, or rates of change.
2. To compare two series which differ widely in amount. Semi-logarithmic paper offers an excellent means of graphically portraying a relationship between two series whose actual figures vary so greatly that arithmetic paper does not permit of a satisfactory comparison. The relative value of each type of paper for such a purpose may be seen in Fig. 48 (Graphs *a* and *b*), below.

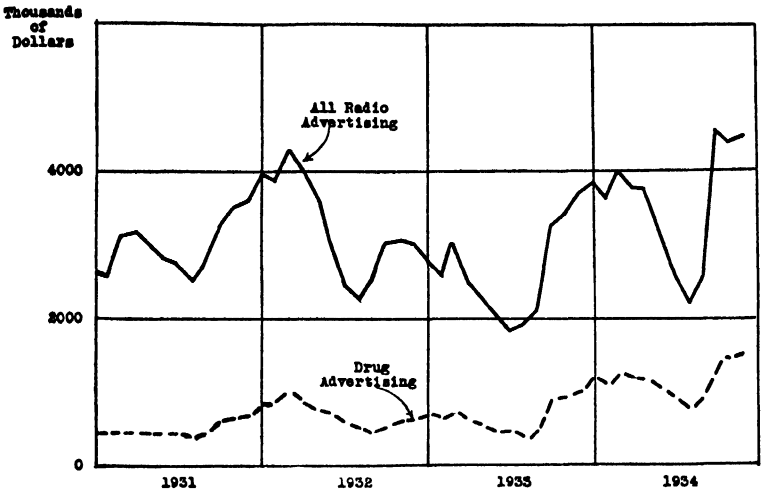
How to Construct a Logarithmic Chart.

1. Prepare a grid or background ruling.

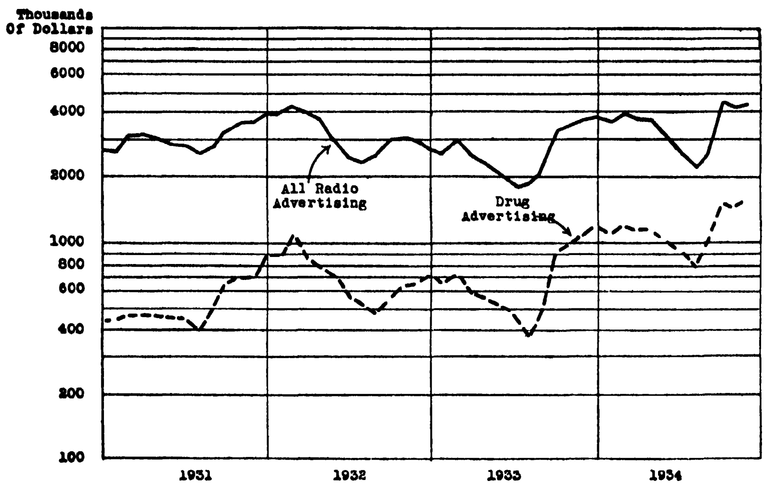
As indicated above, the background rulings on a logarithmic graph are not equally spaced, with the exception of semi-logarithmic graphs, in which case one axis has an arithmetic ruling.

An example of the logarithmic scale shown on p, 78

[4] An arithmetic progression is a series in which the values increase by a constant difference (2, 4, 6, 8, 10, etc.) while in a geometric progression the values are increased by a constant multiplier (2, 4, 8, 16, 32, etc.).



Graph A



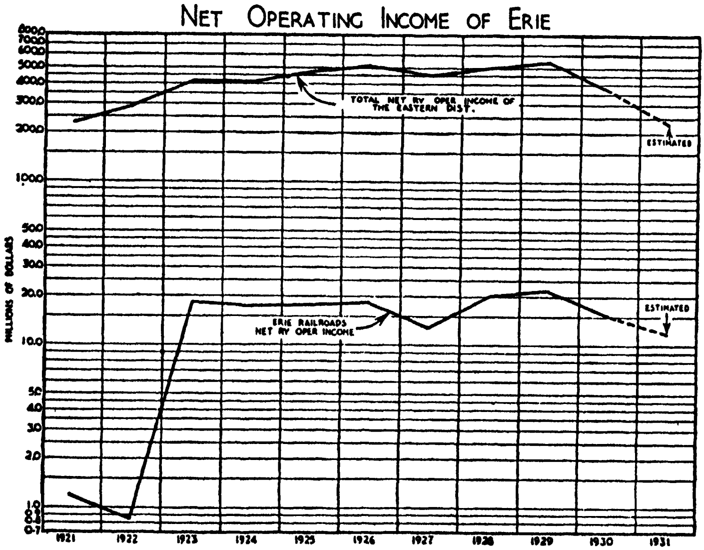
Graph B

Source: Survey of Current Business, United States Department of Commerce

FIG. 48. DRUG AND TOTAL RADIO ADVERTISING (COST OF FACILITIES) IN THE UNITED STATES, 1931-1934
(ARITHMETIC AND SEMI-LOGARITHMIC GRIDS)

indicates that the scale repeats itself in a given cycle, and that each of these cycles (also called phases or tiers) begins with a different power of ten, thus $10^0=1$, $10^1=10$, $10^2=100$, $10^3=1000$, etc., or multiples thereof. Therefore, the first step is to decide on the number of cycles necessary to include all of the data.

The construction of a logarithmic scale can be accomplished with the aid of the scale on a slide rule, by means



From "Magazine of Wall Street," November 14, 1934

FIG. 49. SEMI-LOGARITHMIC GRAPH*

of converting an arithmetic scale, as shown above, by making use of printed sheets, or by using a flexible scale sheet.

2. The data is then plotted. The position of the various points is determined by reference to the two scales. *It is not necessary to look up the logarithms of the original values.* They may be plotted directly.

* In this instance the scale caption, millions of dollars, was printed vertically on the graph due to a lack of space. Although vertically-printed scale captions are often used, they are more difficult to read than if placed horizontally. The latter procedure, therefore, is more advisable on graphs.

Construction of a Logarithmic Scale. A logarithmic scale of any desired size can be readily constructed with the aid of a set of logarithm tables.[5] The size of a logarithmic scale is determined by the height of the cycle. Thus, if it is desired to construct a logarithmic scale made up of 5-inch cycles, a 5-inch distance is marked off on the axis. Each value on the logarithmic scale is then measured off on this axis, in proportion to the logarithms of the numbers.

Each cycle begins with a power of 10 (that is 1, 10, 100, 1,000, etc.). The distance between the natural scale values at the beginning of each cycle (1, 10, 100, etc.) in terms of logarithms will be 1. For example, if a cycle begins with 1 and ends with 10 the difference in logarithms between these numbers will be 1, since the logarithm of 1 is 0.0000 and the logarithm of 10 is 1.0000. If a cycle begins with 100 and the next cycle begins with 1,000, the difference in logarithms will again be 1, since the log of 100 is 2.0000 and that of 1,000 is 3.0000.

In terms of logarithms (rather than the natural scale numbers) the height of a cycle may be equated to 1. The location of each scale value can then be determined by using a table of logarithms. The logarithms necessary for the construction of a logarithmic scale, with a 5-inch cycle, beginning with one, are as follows:

Number	*Logarithm*
1	0.00000
2	0.30103
3	0.47712
4	0.60206
5	0.69897
6	0 77815
7	0.84510
8	0 90309
9	0 95424
10	1 00000

The distance from 1 (the number beginning the cycle) to 10 (the number ending the cycle) is 5 inches. This distance in logarithms is equal to 1.0000. In order to locate the natural num-

[5] Common or Briggs logarithms (to the base 10) are used for this purpose.

ber 2 on the logarithmic scale the percentage of the total distance which its logarithm represents is computed. The logarithm of 2 is 0.30103, but if the distance from 1 to 10 on the scale is 5 inches, and its logarithm is 1.000, the distance from 1 to 2 (in terms of logarithms 0.30103—0.00000 or 0.30103) will be 0.30103 of 5 inches, or 1.50515 inches. Thus 3 (the logarithm of which is 0.47712) will be 0.47712 times 5, or 2.38560 inches from 1.

In similar manner, the distance of each value from 1 (the beginning of the cycle) can be computed by multiplying its logarithm by the height of the cycle. This computation for 3-, 4-, 5-, and 6-inch cycles has been completely worked out below.

TABLE FOR CONSTRUCTION OF 3", 4", 5", AND 6-INCH CYCLES FOR LOGARITHMIC SCALES*

Number	Logarithm	Distance from Beginning of Cycle in Inches			
		3-inch Cycle	4-inch Cycle	5-inch Cycle	6-inch Cycle
1	0.00000	0.00	0.00	0.00	0.00
2	0.30103	0 90	1 20	1.51	1.81
3	0.47712	1.43	1.91	2.39	2.86
4	0.60206	1.81	2.41	3.01	3.61
5	0.69897	2.10	2 80	3 49	4.19
6	0.77815	2.33	3.11	3.89	4.67
7	0 84510	2.54	3.38	4.23	5 07
8	0.90309	2 71	3.61	4.52	5 42
9	0 95424	2 86	3.82	4.77	5.73
10	1.00000	3 00	4.00	5.00	6.00

* These values shown to the nearest hundredth of an inch.

Since each cycle is a replica in form of the first cycle, the remaining cycles in a multiple cycle logarithmic scale may be constructed merely by repeating the measurements for the first cycle. For a high degree of accuracy in constructing a logarithmic scale, a machinist's scale calibrated in one-hundredths of an inch or an engineer's scale calibrated in fiftieths of an inch should be used.

Construction of a Flexible Scale. In constructing a number of logarithmic scales of varying sizes, a flexible scale will save considerable time. Such a scale can be readily constructed by use of the technique outlined above.

A sturdy quality of paper or, still better, tracing cloth, should be used for a flexible scale.[6] The first step in the preparation of a flexible scale is to draw a light horizontal guide line which will serve to locate the center of the scale. This line is later erased. A 4-inch logarithmic scale is constructed on an axis perpendicular to the guide line, with 2 inches of the scale above the line and 2 inches below, at the left side of the sheet. The method described above for constructing a logarithmic scale is used for this purpose.

A 6-inch scale may then be constructed on an axis also perpendicular to the horizontal guide line at a distance of from 4 to 5 inches from the first scale. Half of this scale (3 inches) should be below the guide line and half above.

The points on the two scales for the same values (1 and 1, 2 and 2, etc.) may then be joined by straight lines. Vertical lines are drawn across these lines at appropriate intervals to indicate scales of different sizes. If a sheet of about 12x20 inches has been used, the flexible scale can be used to construct scales ranging from a 1-inch cycle to a 10-inch cycle by folding the scale at the horizontal line, indicating the desired size of the scale and then marking off the divisions on the new axis.

A pre-ruled printed 2-cycle semi-logarithmic grid is shown in Fig. 51.

This printed sheet can be obtained in a wide variety of rulings to meet any ordinary need.

If the scale is correctly constructed, constant percentage increases will be allotted the same distances. Thus, distances between 2 and 4, 4 and 8, 8 and 16 (100-per-cent increases) will be

[6] Poor quality paper will tear as a result of the constant folding to which flexible scale paper is subject.

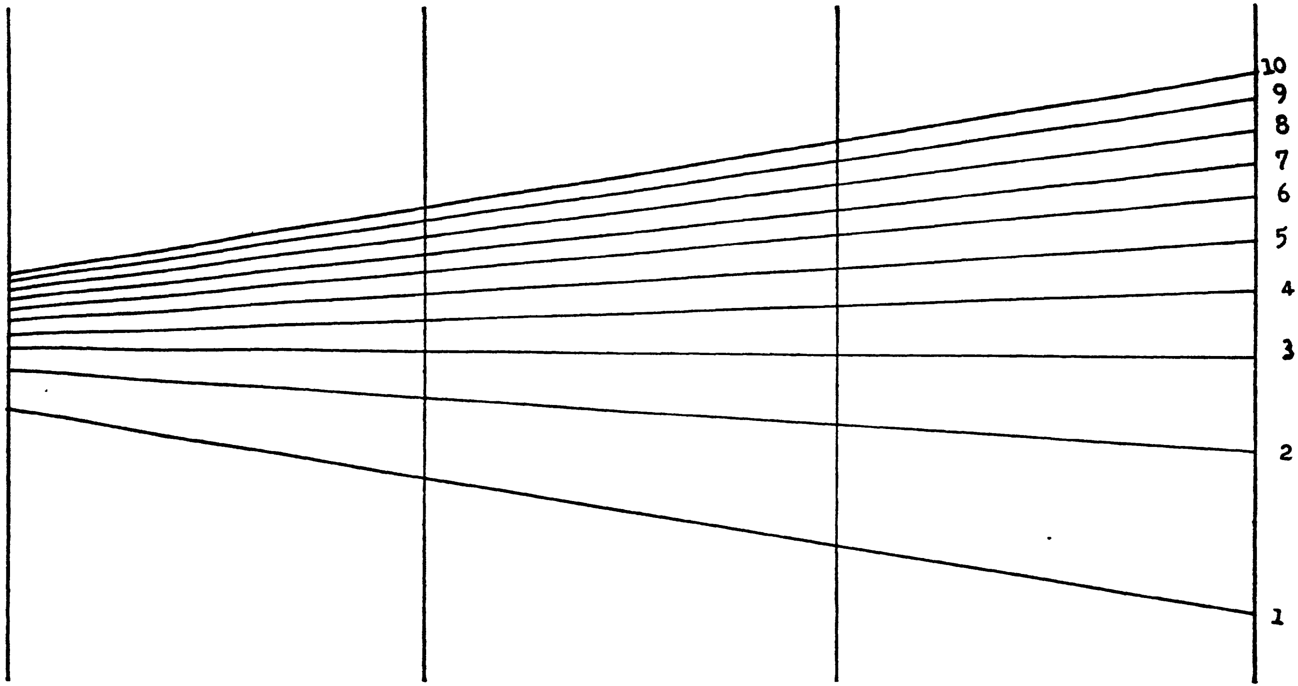


FIG. 50. FLEXIBLE LOGARITHMIC SCALE (REDUCED)

the same, while the distances between 2 and 3, 8 and 12, 16 and 24 (50-per-cent increase) will also be constant in size.

The scale placed on the grid is determined by the number of cycles used. In preparing the arithmetic scale on semi-logarithmic paper, the same rules are applied that were used in the preparation of the arithmetically scaled graph. It is important not to indicate too many scale values nor to use too many coordinate or background lines. These rules are particularly significant in the preparation of logarithmic paper, since toward the top of each cycle the spacing is sharply contracted.

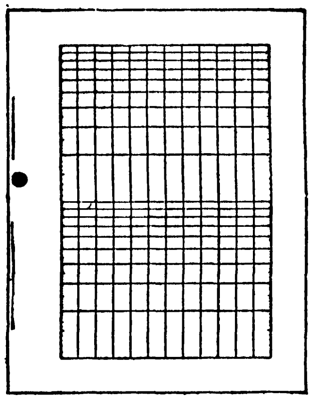


FIG. 51. PRINTED SEMI-LOGARITHMIC SHEET

The *scale captions, table, source, footnotes,* etc., are prepared in the same manner as for the arithmetic graph (pages 46 to 54). If it is more convenient, the logarithmic scale, which ordinarily begins with a power of 10, may be multiplied by any whole or decimal value.

The original scale multiplied by 2, 5, and 8 is shown in Fig. 52. An even figure should be used as a multiplier, since odd figures, such as 3, 7, etc., make it difficult to interpolate on the scale.

Per-cent Scales. Since equal percentage changes are allotted equal distances on a logarithmic chart, it is possible to construct scales

which may be used to determine percentage changes in the data plotted. These scales are (1) a scale to determine increase in per cent, and (2) a scale to determine decrease in per cent.

The preparation of the scale of percentage increase is based on the fact that on a logarithmic scale a given percentage change, regardless of the absolute quantity on which it is based, will al-

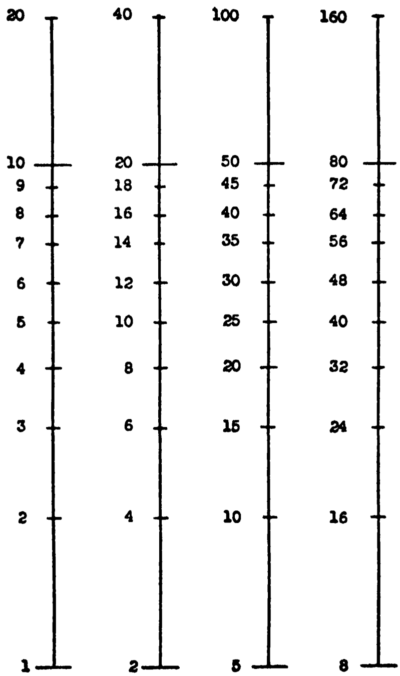


FIG. 52. LOGARITHMIC SCALES

ways have the same distance. Thus, the distance from 2 to 4 (100-per-cent increase) will be the same as that from 12 to 24, 80 to 160, 300 to 600, etc. (also 100-per-cent changes).

In constructing a rule or scale of percentage increase a strip of paper of one half to one inch wide and one cycle in height is prepared. On the strip, the distance from the beginning to the end of the cycle is indicated. This distance will represent a 900-

per-cent increase, since a cycle begins with 1 (or 10 or 100, etc.) and the next cycle begins with 10 (or 100 or 1,000, etc.). This distance between any two points on the scale will represent a 900-per-cent increase. An 800-per-cent increase is determined by marking off on the scale the distance from 1 to 9 (or 10 to 90, etc.). The same procedure is followed for all the other percentages desired.

However, since 900 per cent is represented by the distance from 1 to 10 (the length of the cycle), 800 per cent by the distance from 1 to 9, etc., these rulings match the logarithmic scale. The construction of a scale of percentage increase can be accomplished by ruling off divisions to match the rulings on a logarithmic cycle and replacing the original figures with percentages. Ten is replaced by 900 per cent, 9 by 800 per cent, 8 by 700 per cent, etc., until 1 has been replaced by 0 per cent. In order to determine the per cent of increase on a curve plotted on logarithmic paper, the percentage-increase scale of the same size as the cycle of the logarithmic paper is compared with the distance of the rise of the curve. A scale of percentage increase is shown in Fig. 53 (scale A).

A scale of percentage decrease is similarly constructed. Since, again, a given per cent decrease always is allotted a fixed distance on the scale, a scale of percentage decreases may be prepared.

The distance from 10 (the top of the cycle) to 1 (at the bottom of the cycle) is a 90-per-cent decrease (a distance from 10 to 1 is a 90-per-cent decrease). The distance from 10 to 2 is an 80-per-cent decrease, from 10 to 3 a 70-per-cent decrease. These distances, and, therefore, the markings on the scale, correspond to the logarithmic scale divisions of the cycle. However, in this case the numbering of the divisions is inverted. Thus 10 will be marked —0 per cent, 9 marked —10 per cent, 8 marked —20 per cent, etc. The scale divisions and their corresponding divisions on a scale of per cent decrease are shown below:

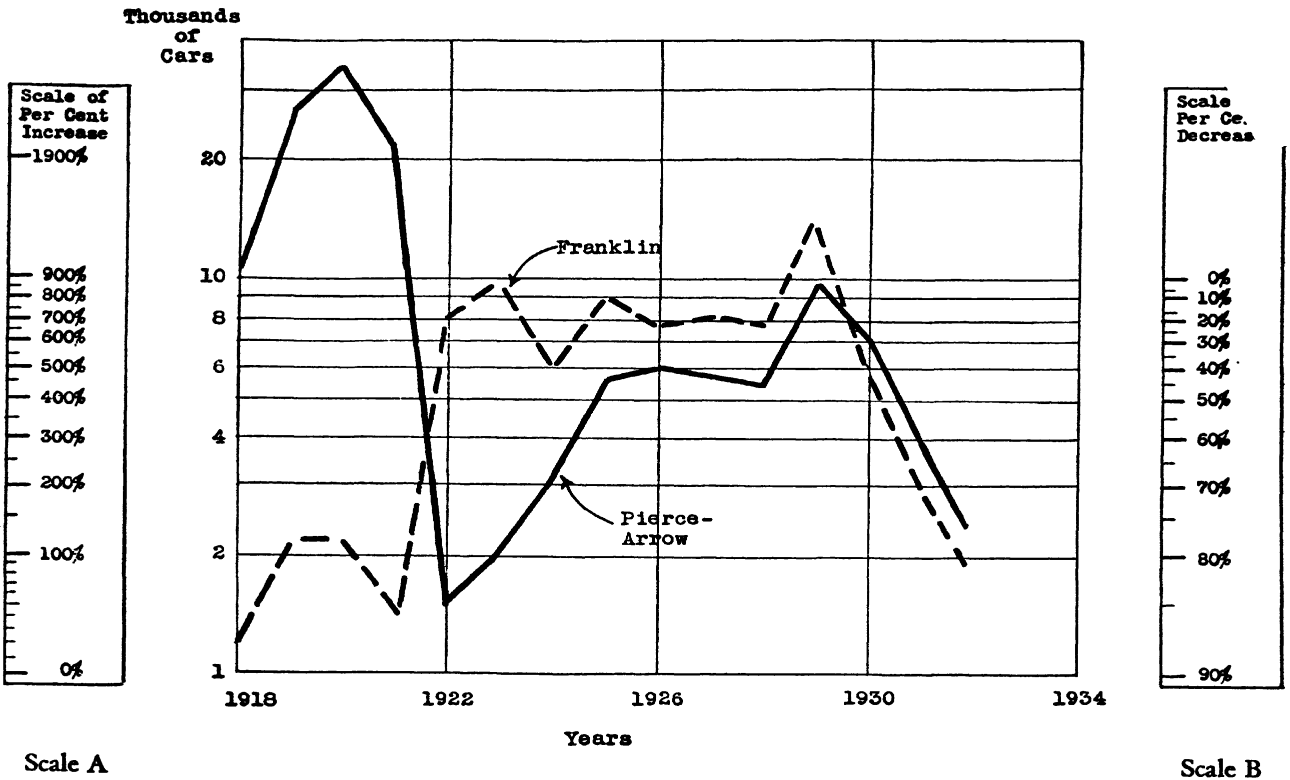


FIG. 53. WHOLESALE SALES OF PIERCE-ARROW AND FRANKLIN AUTOMOBILES, 1918-1934 (SEMI-LOGARITHMIC GRID WITH SCALES OF PER CENT

Scale Division	*Per Cent Decrease*
10	− 0%
9	−10%
8	−20%
7	−30%
6	−40%
5	−50%
4	−60%
3	−70%
2	−80%
1	−90%

A scale of percentage decrease is shown in Fig. 53 (Scale B). A per-cent decrease on a curve, plotted on logarithmic paper, having a cycle of the same size as the scale, is measured by comparing the scale with the distance covered by the curve in its decline and reading the scale *downward.*

Frequency Distributions on Logarithmic Paper. Frequency distributions[7] may be classified into two major groups, those which assume the form of symmetrical curves, or distributions which form asymmetrical curves.

The normal curve is the basic form of symmetrical curve, although there are many other forms of symmetrical curves.[8] However, the most usual type of curve is the asymmetrical one.

Many curves which appear to be asymmetrical when plotted on an arithmetic grid become symmetrical in form when plotted on a logarithmic grid. In this case, a semi-logarithmic scale is generally used, with a logarithmic scale on the X axis and an arithmetic scale on the Y axis.

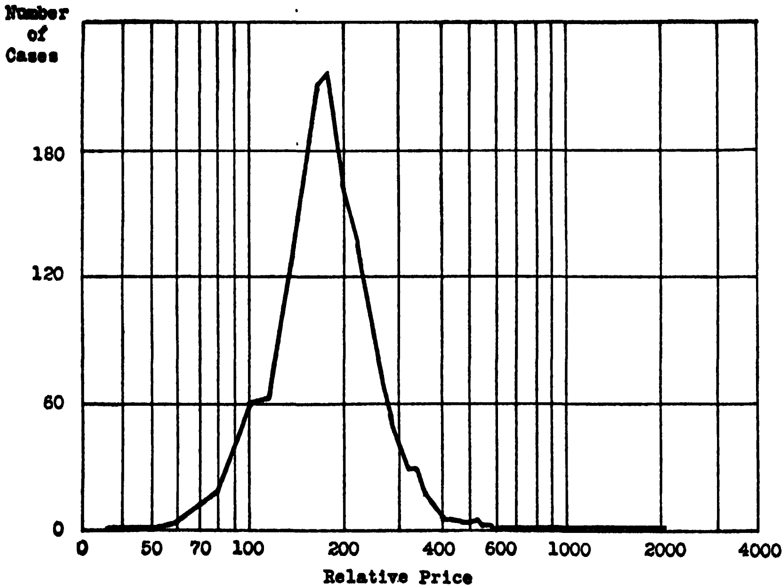
Other Rulings. There are various other types of special rulings which are used infrequently, some of which are discussed below.

Polar Coordinate Paper. Paper of this type consists of a series of equally-spaced concentric circles with a series of equally-spaced radii emanating from the center of the circle.

[7] See p. 45.

[8] For a discussion of normal curves see Arkin, Herbert, and Colton, Raymond R., *Statistical Methods,* 4th ed., chap. XII. Barnes and Noble, New York, 1939.

Two variables may be treated at once on this type of paper by using the distance from the center (as indicated by the concentric circles) for one scale, and the position in the circle (as indicated by the radii) for the other scale.[9]



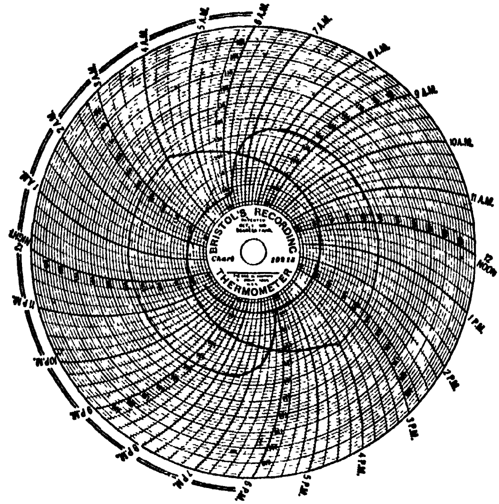
After F. C. Mills in "Statistical Methods"

FIG. 54. DISTRIBUTION OF RELATIVE PRICES OF 1437 COMMODITIES IN THE UNITED STATES IN 1918 (AVERAGE PRICES JUNE-JULY 1914 = 100) (SEMI-LOGARITHMIC GRID)

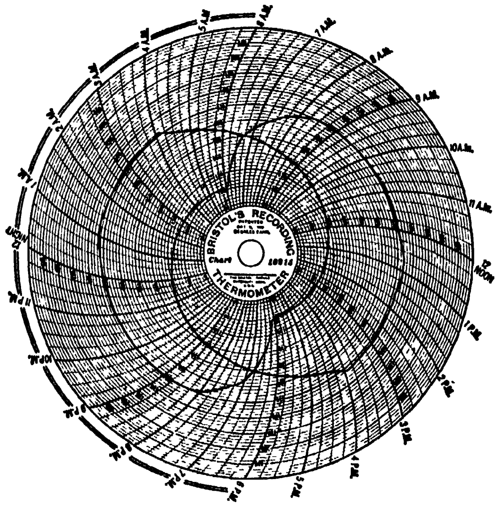
The Polar Coordinate graph is used frequently on recording instruments in which the graph, by revolving, indicates the position in time (as demarcated by the radii), while a recording point varies in distance from the center to make the recording.

A graph of this type, showing the recording for varnish baking at 280° F., and for japan baking at 355° F., is illustrated in Fig. 55.

[9] These radii may be straight or curved.



Varnish oven



Japanning oven

Recording charts for two consecutive days showing the temperature cycle at 280 deg. F. for varnish baking and 355 deg. F. for japan baking

From "Industrial Gas," October 1931

FIG. 55. A RECORDING POLAR COORDINATE GRAPH

Trilinear Diagrams. Trilinear graphs are used to show the relation between *three* variables at the same time. The graph is constructed in the form of an equilateral triangle, with scales for each of the three variables on the three sides of the triangle. A number of curves may be placed on this graph to show the relation between the three variables for various situations or requirements.

The trilinear diagram is generally used to present percentage relationships, and therefore the scales on each of its three sides are generally in terms of per cent. This type of graph is usually used to present the variations in three components of a total.

The trilinear grid may be constructed by drawing an equilateral triangle (equal sides) and arranging a scale for each of the sides. The three sides may be divided into 10 parts (one part for each 10 per cent) and each of these divisions joined to the corresponding divisions of the other axes. The coordinate lines indicating the scale divisions for each of the axes run *parallel* to the side of the triangle. Thus, the coordinates indicating the scale divisions for one axis (A) will be horizontal and equally spaced, while those for the other axes will be parallel to their respective sides, or diagonal to the base at the triangle. The scales for the B and C axes run *down*, while that at the A axis runs *up*. Reading and plotting values on this type of graph are somewhat difficult for the uninitiated because of the peculiar arrangement of the scales.

The location of a point may be determined from the value on its A scale in the following manner: When the value of that point is, for example, 50 per cent, and if there are but 10 divisions (one for each 10 per cent), the value can be located by counting 5 divisions above the horizontal side of the triangle. The scales can be said to run along a vertical axis perpendicular to the axis under consideration. The values for the scales for the other axes may also be determined by counting divisions *away* from the proper axis. The numbers running through the grid in Fig. 56

are the scale values. The scale caption for the vertical axis may be placed at the top of their respective scales. A common difficulty encountered in trying to interpret the scale is that many, seeing the scale caption along the side of the triangle, count the number of divisions along that side. The divisions should be counted

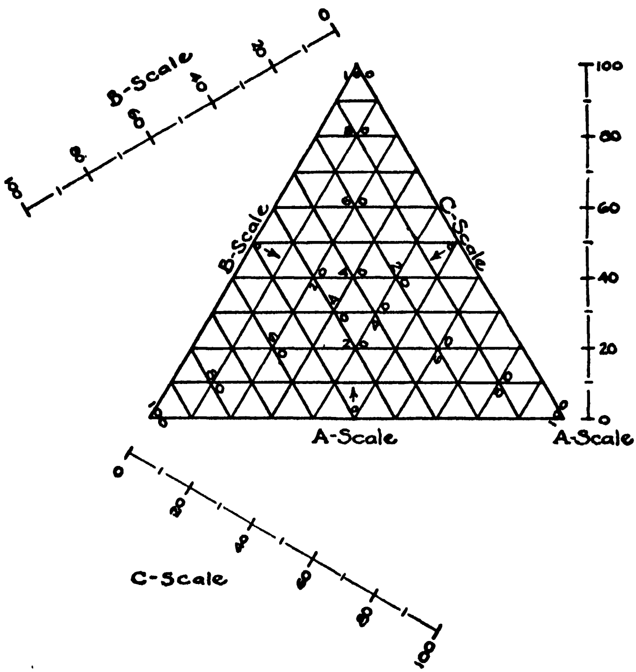


FIG. 56. TRILINEAR DIAGRAM—POSITION OF THE THREE SCALES

along the *axes* and not along the side of the triangle. The true positions of the scale axes are shown by the dashed lines in Fig. 56.

Since the scale captions for each of the axes are placed in many different positions, trilinear diagrams are often difficult to read and understand.

If these scale captions are placed at the bottom of the axes (and therefore at the center of the sides of the triangle) a point

may be readily located by following the two sets of diagonal rulings. To locate a point, given its A, B, and C values, it is only necessary to follow the diagonal lines running from the B side of the triangle and count off a number of divisions equal to the per cent given. The coordinate at that point is noted. The same procedure may then be followed for the C value. Divisions may be counted off along the other diagonal lines until a sufficient number have been included to indicate the required per cent. The point of intersection of the coordinate at this point with the previously noted coordinate is the location of the plotted point. It is not necessary to use the other value for plotting, since any two values will determine the third (for $A + B + C = 100$ per cent). This other value may be used, however, to determine the accuracy of the plotting.

Since there are three scales, this type of graph can handle three variables simultaneously. It can, therefore, be widely used for graphic analysis wherever there are three components constituting the whole, or 100 per cent. Variations in all of these quantities can be shown simultaneously by the position of the plotted points. The advance of any plotted point along the axes (which are either vertical for the A axis or diagonal to the base of the triangle for the B and C scales) indicates an increase in the per cent of that variable. It is obvious that an increase in a variable must be obtained only at the expense of the others, since they all total to 100 per cent.

Because of the ability to handle three variables simultaneously, this type of graph has been used for presenting business and scientific data of various types.

An example of its use may be seen in the analysis of the income of a business. The income is disbursed in the form of expenses, which may be divided into overhead costs, variable costs, and profits. The construction of a trilinear diagram for this type of data as shown in Table V is outlined below.

TABLE V

DISBURSEMENT OF INCOME OF ACME CO., 1932–1934

Years	*Overhead*	*Variable Costs*	*Gross Profit*
1932	40%	55%	5%
1933	40%	50%	10%
1934	35%	45%	20%

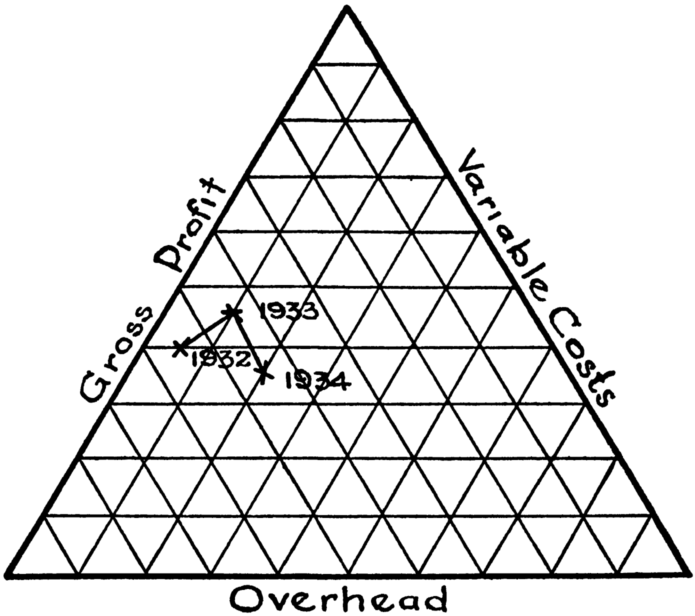
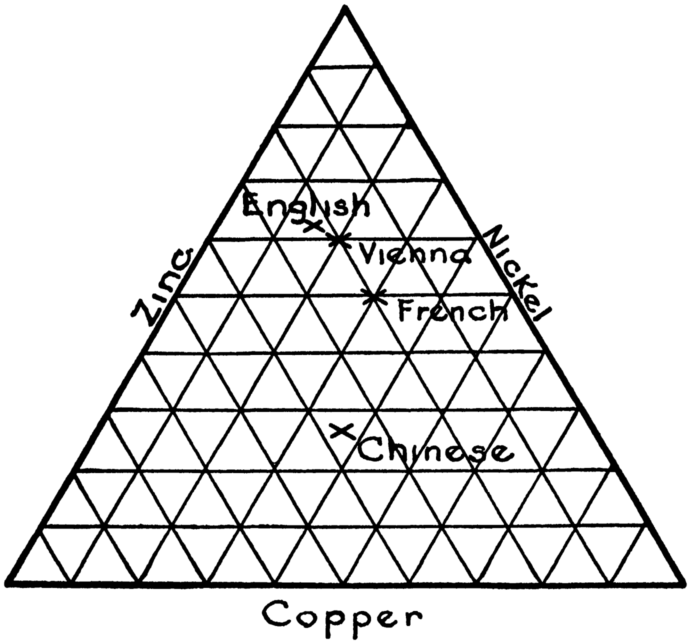


FIG. 57. DISBURSEMENT OF INCOME OF ACME CO., 1932-1934 (TRILINEAR DIAGRAM)

A somewhat different use of the trilinear graph is to show the percentage composition of different substances. The percentage composition of German-silver alloys of different kinds is shown in Fig. 58.

Other Special Rulings. A geometric progression will plot as a straight line on logarithmic paper. It is frequently desirable to convert various other types of curves into straight lines. This may be accomplished for many curves by resorting to appropriate rulings.

Probability Ruling. A useful type of ruling is that which is used to straighten out a cumulated normal probability curve.[10] Specially ruled paper of this type may be purchased in printed form. A sheet with probability ruling and with a cumulated probability curve plotted on it is shown below.



Source: Henley's Book of Formulas 1935, p. 70

FIG. 58. PERCENTAGE COMPOSITION OF GERMAN SILVER ALLOYS (TRILINEAR DIAGRAM)

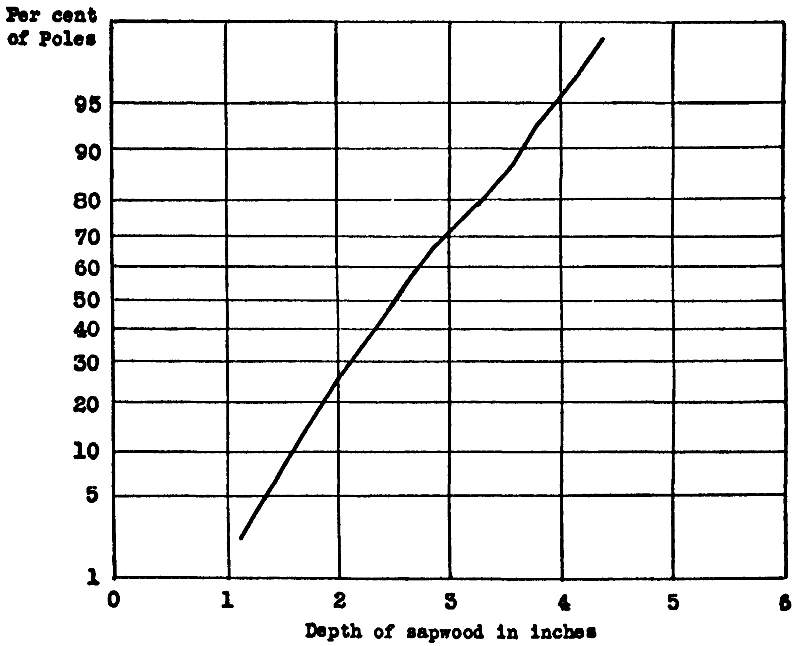
The distribution is first arranged in the form of an ogive and the frequencies converted into per cent of total cases. This percentage ogive may then be plotted on the probability grid. The ruling on this type of grid is arranged so as to convert the ogive from its usual "S" shape into the form of a straight line.

A glance at the chart will then tell the degree of departure from symmetry of the distribution plotted. Since the normal

[10] A cumulative curve is known technically as an ogive.

probability curve is often the result of the operation of natural phenomena, departure from this form is often indicative of extraordinary circumstances worthy of investigation.

Fig. 59 shows the distribution of the depth of sapwood in telephone poles. An analysis of these data is of commercial importance. Departure from normality is a matter for investigation.



Source: Shewhart, W. A., "Economic Control of Quality of Production"
FIG. 59. DISTRIBUTION OF DEPTH OF SAPWOOD IN 1370 TELEPHONE POLES—PROBABILITY RULING (LESS THAN OGIVE)

It is to be noted that the frequencies are plotted at the lower limits of the class intervals for the "less than" and the "and over" forms of ogive.

Numerous other rulings are available for similar purposes. A discussion of highly specialized rulings is beyond the scope of this volume, since such rulings are of interest to only a very limited number of persons.

Rules to Avoid Common Errors Found in Ratio Charts.

1. A zero line cannot be used in constructing a logarithmic scale, since the logarithm of zero is minus infinity. Since zero is never indicated, a scale break is unnecessary on a logarithmic graph.
2. On a semi-logarithmic graph the arithmetic scale is used for the time scale. Time, the independent variable, appears on the same axis (the X axis) as the arithmetic ruling on the semi-logarithmic chart since time is always in the form of an arithmetic progression.
3. The scale numbers should be placed directly next to the background line to which they apply.
4. Each cycle on a logarithmic scale should begin with a power of 10—that is, the cycles should begin with 1, 10, 100, 1,000, etc.
5. Logarithms should not be placed on a logarithmic scale. The arithmetic equivalents of the logarithms should be used. In this way it will be unnecessary to find the logarithms of a number to plot the number on a logarithmic graph.
6. A minimum number of background rulings should be used on ratio charts.

CHAPTER VII

THE BAR CHART

Types of Bar Charts—When to Use the Bar Chart—How to Construct a Bar Chart—The Title—The Source—Footnotes—Special Problems in Construction—Absolute-Component Part Bar Chart—Relative-Simple Bar Chart—Relative-Component Bar Chart—The Single Bar Chart—Multiple-Unit Bar Charts—Compound Bar Charts—Loss and Gain Bar Charts—Pictorial Bar Charts—The Cosmograph—Rules to Avoid Common Errors Found in Bar Charts.

THE bar chart visually contrasts quantities by a comparison of bars of varying length. The length of each bar is determined by the size of the values. The width of each bar is constant. The bars in a bar chart may be presented horizontally or vertically, the vertical bar chart being used when an element of time is involved.

A simple bar chart, presenting pictorially the fluctuations in exports from the United States for the years 1920, 1925, and 1930, is shown in Fig. 60.

Types of Bar Charts. Bar charts classified according to the manner in which they present information are:

1. Bar charts presenting *absolute values* (*i.e.*, original units of the data, as tons, dollars, students, etc.).
2. Bar charts presenting values in *percentage* or relative form.

In turn, these two major types may be subdivided according to whether the contrast is made merely between totals or an attempt is made to compare the component parts of the totals as

well. Thus, in Fig. 61, Graph A, the total sales of the Acme Co. is contrasted for three years, while in Graph B of the same figure, the sales of the territories which go to make up the total sales are contrasted as well.

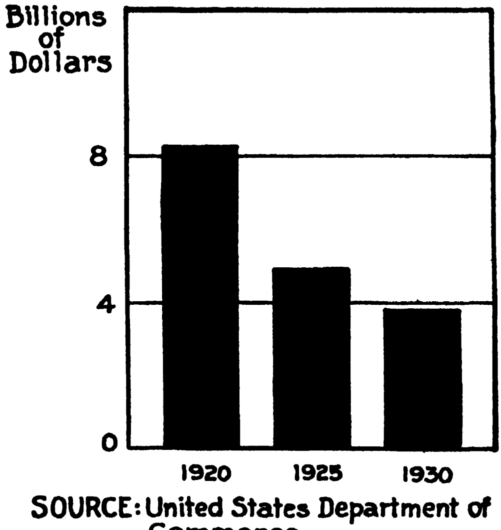
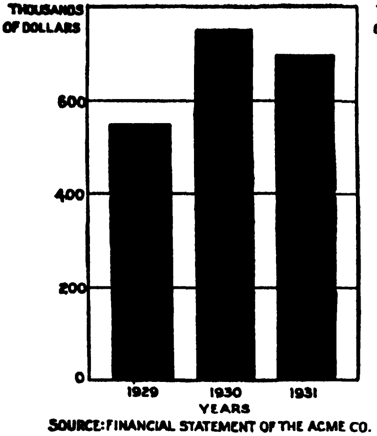


FIG. 60. EXPORTS FROM THE UNITED STATES, 1920, 1925, AND 1930 (BAR CHART)

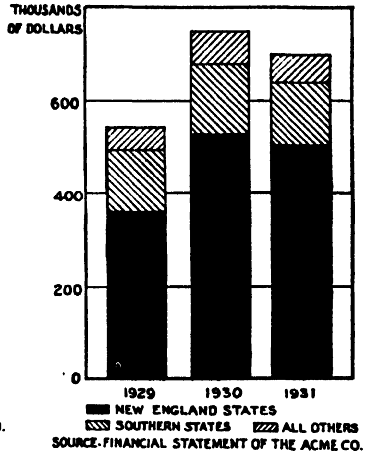
The various types of bar charts may then be outlined as:

I. Absolute bar charts
   a. Simple (see Graph A, Fig. 61)
   b. Subdivided (see Graph B, Fig. 61)
II. Percentage or relative bar charts
   c. Simple (see Graph C, Fig. 61)
   d. Subdivided (see Graph D, Fig. 61)

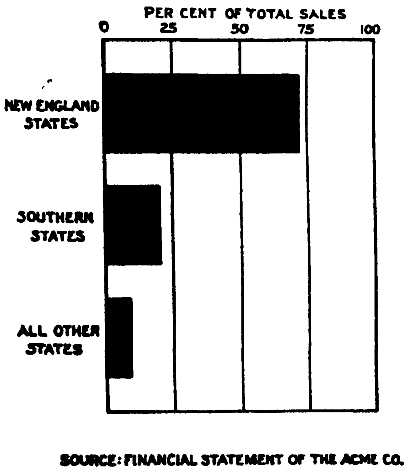
When to Use the Bar Chart. The bar chart is the most effective form of presentation for a comparison of a very limited number of values, generally not more than three or four, or when comparing quantities specified for given places, types, or kinds.



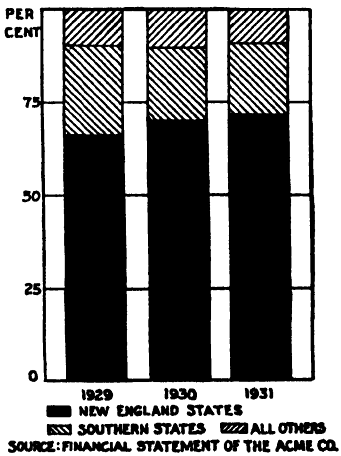
A



B



C



D

FIG. 61. SALES OF THE ACME COMPANY, 1929-1931 (GRAPH C IS FOR 1931 ONLY) (TYPES OF BAR CHARTS)

Where the data for a limited number of quantities relate to places, kinds, types or specifically named items, the only graphs which may be used are the bar chart or the multi-dimensional graphs, such as the area or solid diagram. Area graphs are considerably less effective for the presentation of such data, and for this reason the bar chart should be used.[1]

How to Construct a Bar Chart (Simple Absolute Type).

1. Prepare a *grid* or *background* ruling. The background rulings (coordinate lines) are uniformly spaced (see Fig. 62). This type of ruling is known as the arithmetic grid.[2]

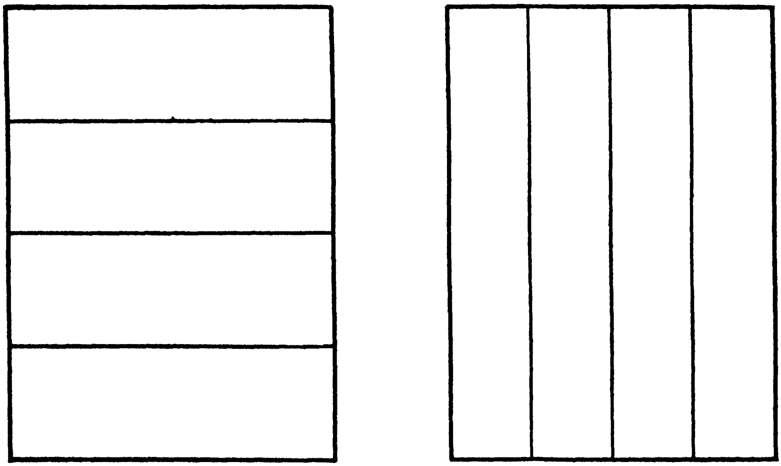


FIG. 62. GRIDS FOR VERTICAL AND HORIZONTAL BAR CHARTS

As few background lines as possible should be used (see page 47). The appearance of the graph will be improved if a special grid is prepared on an unruled sheet of paper rather than on a pre-ruled printed sheet.

2. A suitable *scale* of values is selected and arranged on the side of the grid.

[1] See Chapter VII for an explanation of the relative effectiveness of the area diagram for the presentation of these data.

[2] See page 47.

The grid selected should be such that the largest bar will reach nearly to the top of the grid. It must range from zero to somewhat more than the maximum value.

The intervals of the scale (*i.e.,* the values indicated along the axis) should be in round numbers (such as 2, 4, 10, 20, 25, 50, 100, etc.) rather than in odd or unusual quantities.

The scale in a vertical bar chart (which is used for time variations) is arranged along the left-hand vertical axis. In the horizontal chart these values are arranged along the top of the graph.

The *labels* for the bars on a vertical chart are placed along the bottom axis directly under the bars; in a horizontal chart, at the left side next to the bars.

A *caption* indicating the units used is placed at the center directly above the scale in the horizontal chart, and at the top of the scale on a vertical chart.

It is not intended that the actual values for the bars be read off from the scale, but rather that a rough visual comparison be made. For this reason it is not necessary to indicate numerous scale divisions or use many background lines. The actual values may be secured from the table which generally accompanies the graph.

3. The bars may now be placed on the grid. The values indicated are located on the scale, and a bar is constructed to a length corresponding to the scale values.

For the best appearance, the bars should be spaced so that there is a distance between them ranging from one-half to a whole width of a bar, with the first bar at a distance of about one-half of the width of a bar from the grid.[3]

[3] In a few instances the bars are placed next to one another with no space between them. The resulting diagram is known as a histogram. For these cases see page 69.

In order to provide a sharp contrast between the bars and the background rulings, the bars are generally filled in with a solid color or with diagonal lines (called cross-hatchings). It is inadvisable to use vertical or horizontal lines for this purpose, since they tend to give rise to optical illusions in which the bars are seemingly elongated or shortened.

In the diagram below, the two bars seem to be of different lengths, although they are exactly the same in length. This familiar optical illusion results when horizontal and vertical lines are used in the bars.

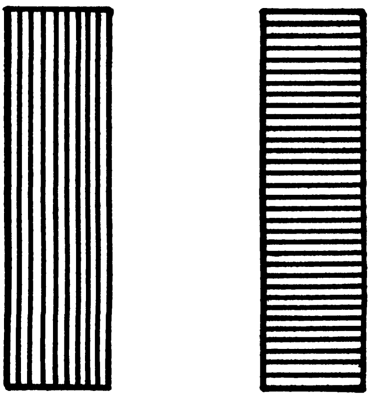


FIG. 63. OPTICAL ILLUSION

Fig. 64 illustrates the graph with the bars placed in their proper places.

The Title. The title of the bar chart appears in the same position, and conforms to the same rules, as the titles for line graphs as discussed more completely in Chapter IV.

The title should be complete and self-explanatory. It should contain the essential elements, consisting of a statement of the nature of the data, the territory covered, and the period of time dealt with. The various elements generally appear in the order indicated above.

As previously noted in the printed graph, the title is placed beneath the chart, while in other graphs it is placed directly above the grid.

The Source. As in all other types of graphs, the source serves a twofold purpose: (1) to stress the authoritativeness and therefore the reliability of the data by indicating some well-known insti-

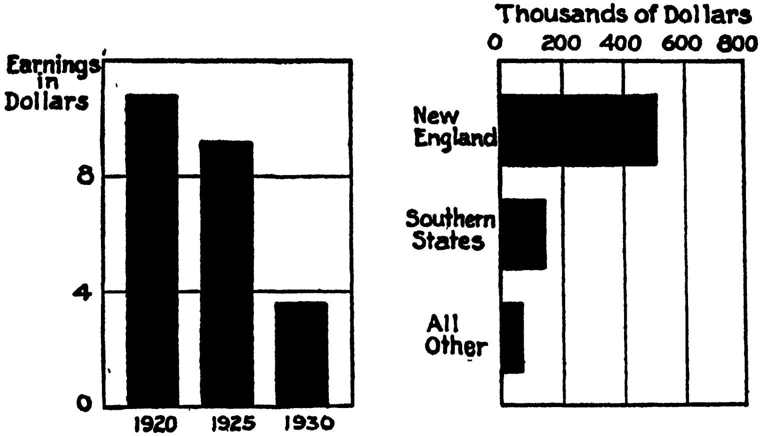


FIG. 64. BARS PLACED ON GRIDS

tution or authority, and, in some instances, (2) to indicate the publication in which further information on the same subject is available. The source is generally placed under the grid, at the left-hand side.

Footnotes. Footnotes explanatory of the special nature of the data, or of special material included or excluded, are placed under the grid at the right-hand side.

Special Problems in Construction. The construction of the various types of bar charts gives rise to special problems which are peculiar to the particular kind of chart.

Absolute-Component Part Bar Chart. In constructing the absolute-component part bar chart (Graph B, Fig 61) the total is

broken down into its various components. The first, and generally the largest, component is laid off on the scale and a bar ruled to that height. The second, or the next largest, segment is then measured off, starting with the *top* of the previous segment. The same result may be accomplished by combining the two figures and placing the top of the bar at that position. This is continued until all of the parts have been included. The second

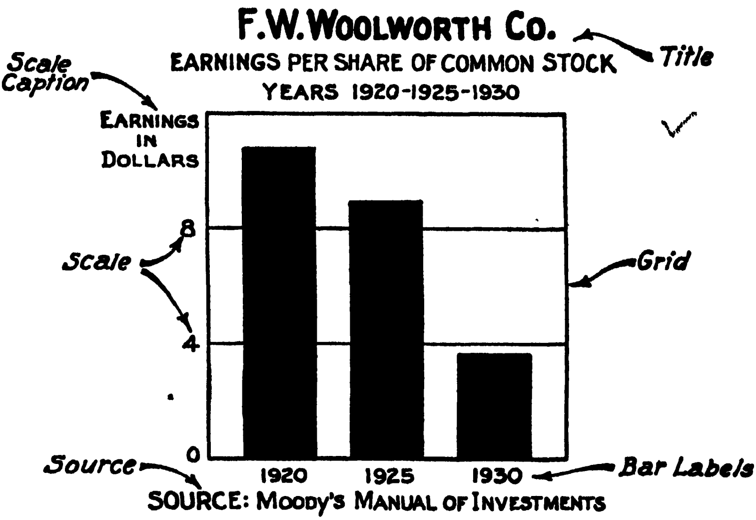


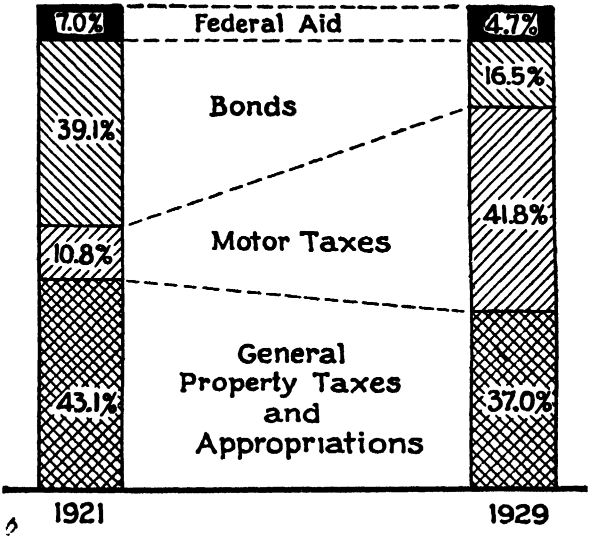
FIG. 65. A MODEL BAR CHART

(and all other bars on the chart) should have each component in the same order as the order of the components chosen in the first bar. A comparison between a given component of two or more bars is now made by comparing the lengths of *those particular sections*, and not the distance of the tops of the segments from the base line.

The component parts of the chart must be identified by means of a key or legend indicating the meaning of the various types of colors or cross-hatchings used. The component parts may also be indicated by means of labels placed next to (or in single bar charts within) the sections.

The key is generally placed directly beneath the grid or, if space is available, on the grid itself.

If the comparison between the various components is of greater importance than the comparison between totals, the grid may be omitted entirely and guide lines drawn in joining the various segments as illustrated in Fig. 66 showing rural highway income in the United States by sources in 1921 and 1929.



Source: National Automobile Chamber of Commerce

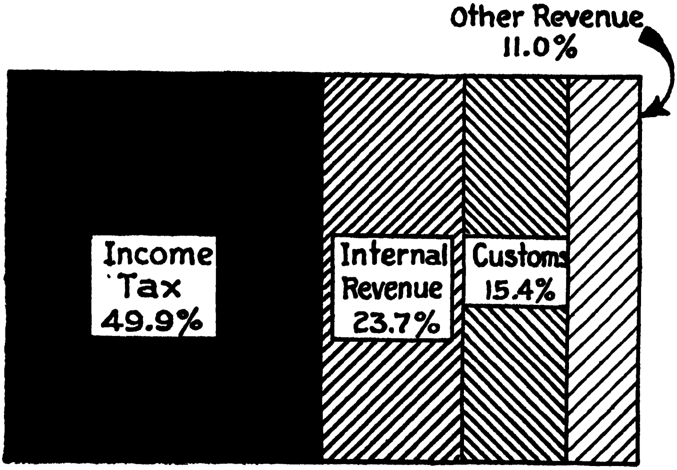
FIG. 66. RURAL HIGHWAY INCOME IN THE UNITED STATES BY SOURCES, 1921 AND 1929 (COMPONENT-PART BAR CHART)

When the contrasted segments are of the same size the guide lines will be parallel; when the same segment of the next bar is larger the lines will diverge, and when smaller, they will converge. In this manner the contrast between the same component at various periods of time or for various places is facilitated.

Relative-Simple Bar Chart. In the relative-simple bar chart (Fig. 61, Graph *C*) the data are presented in percentage form. Each

bar may represent a component part, in which case the total of all the bars will be 100 per cent, or each bar may represent a percentage of the total actual figures for a given part over a period of time.

Relative-Component Part Bar Chart. If the component part type of percentage bar chart is used, each bar will be equal to 100 per cent, since the sum of the various parts must be equal



Source: Annual Report of the Secretary of the Treasury

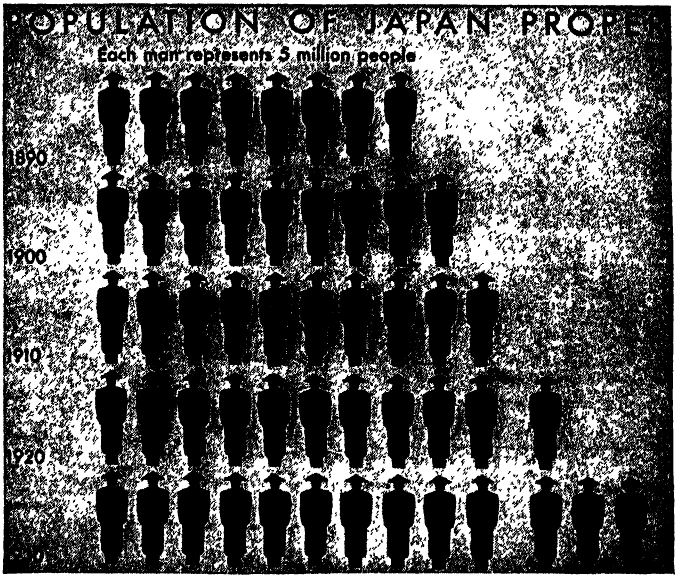
FIG. 67. ORDINARY RECEIPTS OF THE UNITED STATES GOVERNMENT, 1932 (SINGLE BAR CHART)

to the whole. The bars then will be of equal length (see Fig. 61, Graph *D*), while the component segments will vary in size. Even though the sections may not retain the original order of size, they should be set up in the same order in each bar as in the first bar.

The Single Bar Chart. When interest is confined largely to the component parts of a given total for a particular time or place, a single bar is sometimes used without a grid. The values repre-

sented by each segment may be indicated on the bar itself in a "window" or open space or along the top or side of the bar. In Fig. 67, showing the ordinary receipts of the United States Government in 1932, windows were used to represent three values, while the fourth value, in this case other revenue, was written above the bar.

Multiple-Unit Pictorial Bar Chart. Pictorial graphs, with their more interesting and often more effective form of presentation



From "Asia," February 1935, Courtesy of Pictorial Statistics, Inc.

FIG. 68. A MULTIPLE UNIT BAR CHART

than other types of graphic presentation have had an increasing usage in recent years. In fact, the pictorial graph constructed by creating several rows of uniform pictures has become the most popularly used graph in publications.

Each picture, in each of the rows, is used to represent a given number of units. The number of pictorial units will then vary in proportion to the quantities to be compared. The lengths of a

series of such rows may be contrasted to obtain the proportions between the totals to be compared. In addition, an accurate estimate of the actual values can be obtained by counting the number of units in each row.

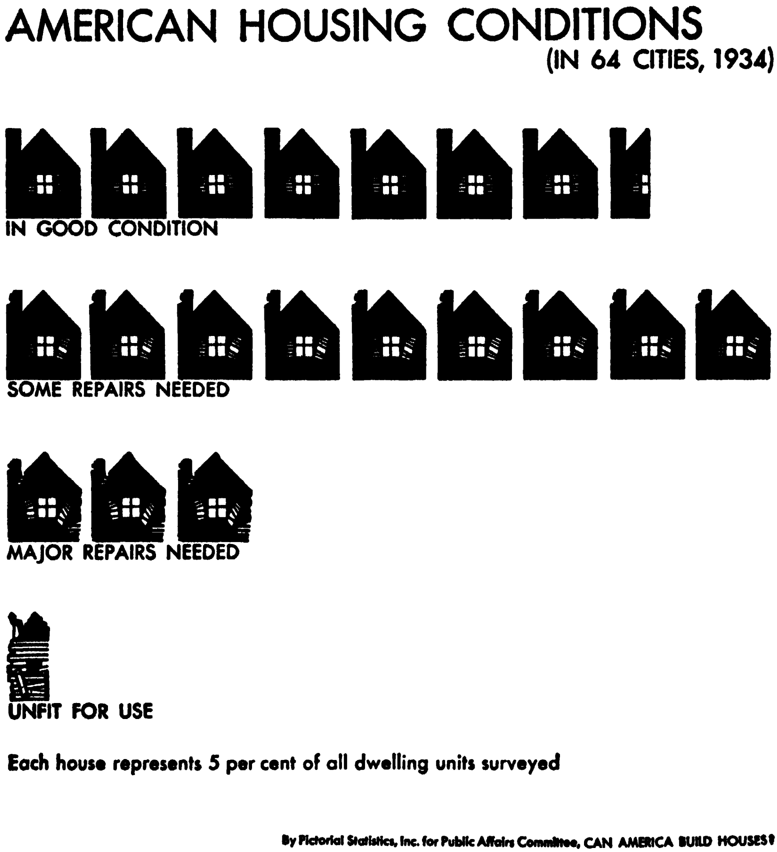


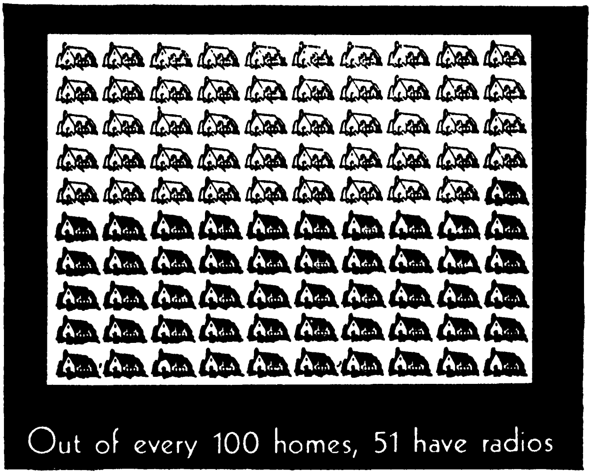
FIG. 69. A MULTIPLE UNIT PICTORIAL BAR CHART

When the pictures used to represent the data are carefully prepared they serve not only to indicate the quantities involved but also to identify the character of the data contrasted. A diagram of this type is shown in Fig. 68. It is to be noted that no grid is necessary. The quantities are indicated by showing in some con-

Reproduced from Modley, Rudolf, "How to Use Pictorial Statistics," Harper & Brothers, New York, 1937

FIG. 69a. TYPES OF SYMBOLS USED IN MULTIPLE UNIT PICTORIAL BAR CHARTS

spicuous position the number of units represented by each picture. In this graph, the population growth of Japan (over a period of years) is shown by a repetition of the conventional figure of an Asiatic in frequency proportionate to the number for each year. Thus, the population of Japan in 1890, about forty millions, was indicated by eight figures, each representing five million people. For each succeeding period the number of figures was increased proportionately.

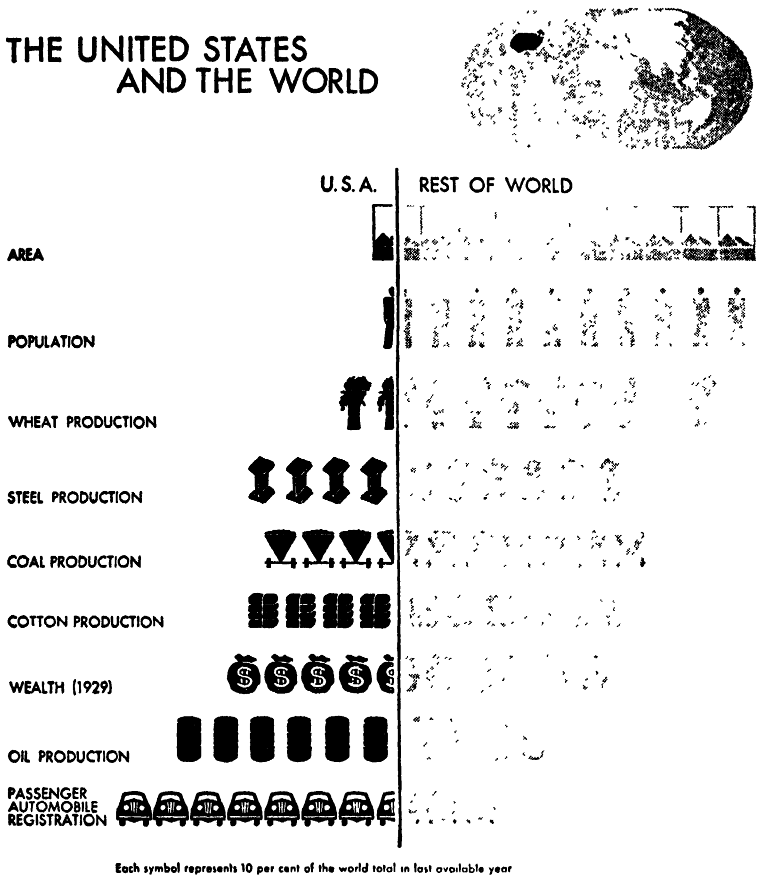


From the Newspapers, Bureau of Advertising, American Newspaper Publishers Association, 1934

FIG. 70. MULTIPLE UNIT PICTORIAL HUNDRED PER CENT BAR CHART

By spacing groups of figures in the multiple unit pictorial bar chart, it is possible to present additional information. Thus in Fig. 68, the growth in population from a base year, in this case 1910, was illustrated by setting apart the figures representing the increase, in 1920 one figure, and in 1930 three figures.

An effective type of pictorial unit for use in graphs of this type is the conventionalized silhouette of a simplified nature. These pictures should be designed to emphasize the peculiar character-



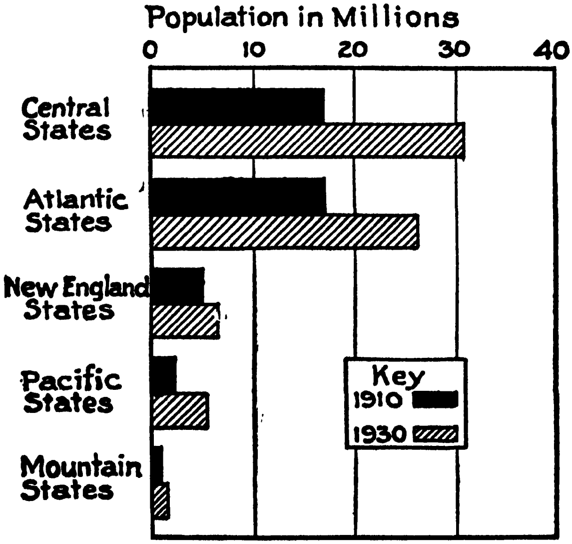
Reproduced from Hacker, Louis M., "The United States, A Graphic History," Modern Age Books, Inc., New York, 1937

FIG. 70a. A MULTIPLE UNIT PICTORIAL BAR CHART

istics of the type of data and presented in such a fashion as to enable the reader to quickly identify the character of the information shown. The use of a variety of such figures may serve to differentiate between the type of data presented in each row of the graph. A graph in which different types of pictorial units distinguish the various types of data presented, is shown in

Fig. 69a. Several pictorial units[4] which typify the kinds of pictures most suitable for presentations of this kind are shown in Fig. 69a.

The most effective multiple unit pictorial bar charts result when from 3 to 6 rows are used and each row includes a maximum of 15 to 20 pictorial units depending upon the type of picture presented. Circumstances may require variations upon this general rule. For instance, where the type of multiple unit bar chart com-



Source: United States Bureau of Census

FIG. 71. URBAN POPULATION OF THE UNITED STATES BY SECTIONS OF THE COUNTRY, 1910 AND 1930 (COMPOUND BAR CHART)

parable to the single bar component part bar chart (see page 110) is used, a single row will be required and this row may sometimes effectively include a larger number of pictorial units. An example of a chart of this type is shown in Fig. 70. The fact that 51 per cent of the homes visited in a special survey had radios was indi-

[4] Several hundred varied types of symbols or pictures useful for the preparation of this type of graph may be purchased in sheet form from Pictorial Statistics Inc., New York City.

cated by showing 100 homes in a block and darkening 51 to indicate the proportion in which radios were found.

A multiple unit pictorial bar chart of the same form as the relative subdivided bar chart is shown in Fig. 70a.

Each pictorial unit in a diagram may be used to represent any given quantity. However, it is most desirable to have each unit equated to a round number since this facilitates the estimating of the totals. Occasionally it is necessary to indicate fractional parts of the basic quantity represented by each unit. Generally, it is not efficient to make use of fractions of pictures other than one half of a unit.

The simplicity of this type of diagram makes it particularly useful for graphic presentations intended for the general public. A pictorial presentation of data may often succeed in presenting complex facts to an uneducated audience more efficiently than other methods. However, the multiple unit pictorial bar chart cannot be used effectively when the data to be compared indicate great differences in the totals to be contrasted. A similar handicap arises when the differences are very small or when there are many totals to be depicted.

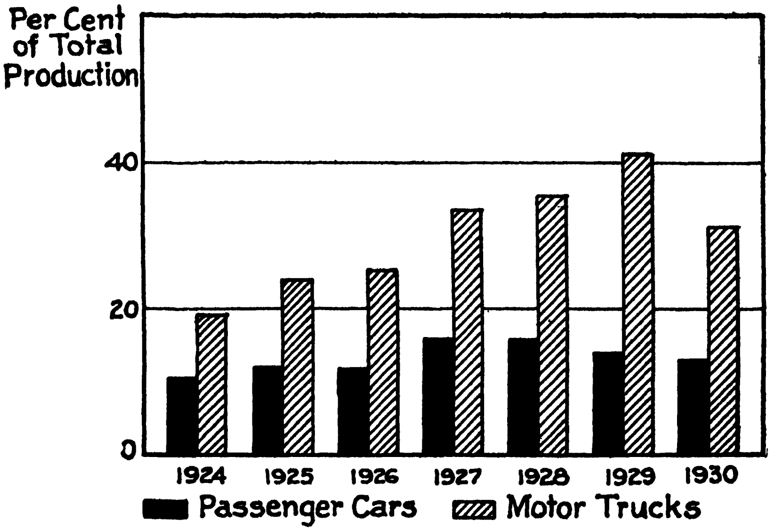
The pictorial nature of the diagrams results in further limiting their use. Since pictures must be employed, it is not possible to present abstract subjects, as for instance data relative to intelligence, efficiency, etc., because they cannot be represented pictorially.

Compound Bar Charts. To contrast the quantities for two periods of time at the same time as the comparison is made between places or types, a double bar chart may be used. In this type of chart two bars are placed without any space between them at the point for each period (see Fig. 71).

The compound bar chart may also be used where a comparison between two allied types of data over a number of periods of time is to be shown or to compare related series for various places or types. In Fig. 72, the exports of passenger cars and motor trucks from 1924-30 were compared.

Loss-and-gain Bar Charts. If losses and gains, or deviations from a normal, are to be presented graphically in bar-chart form, a number of different forms may be used.

The zero line may be placed at the center of the grid, as in Fig. 73, negative values indicated to the left of it and positive values to the right. The bars may now be extended to the left

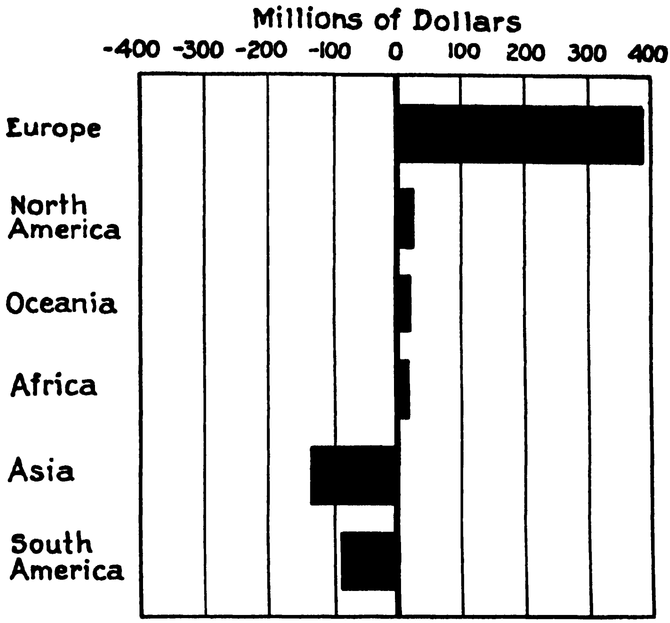


Source: National Automobile Chamber of Commerce

FIG. 72. PERCENTAGE OF AMERICAN AUTOMOBILE PRODUCTION SOLD ABROAD, 1924-1930 (COMPOUND BAR CHART)

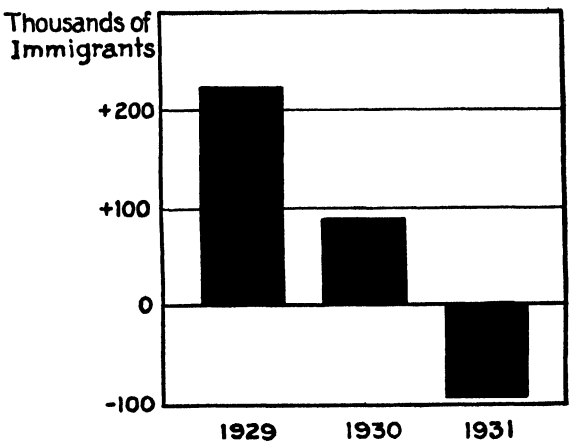
to indicate a loss or negative value, and to the right to indicate a gain or positive value. An example of a loss-and-gain bar chart may be seen in Fig. 73, showing the balance of trade by continents in 1932.

In the vertical bar chart the zero line may be raised, and a negative scale running from the zero line down may be inserted, as shown in Fig. 74. When the time element is to be shown and negative values to be indicated this form of bar chart must be used.



Source: United States Department of Commerce

FIG. 73. BALANCE OF TRADE BY CONTINENTS, 1932 (HORIZONTAL BAR CHART)

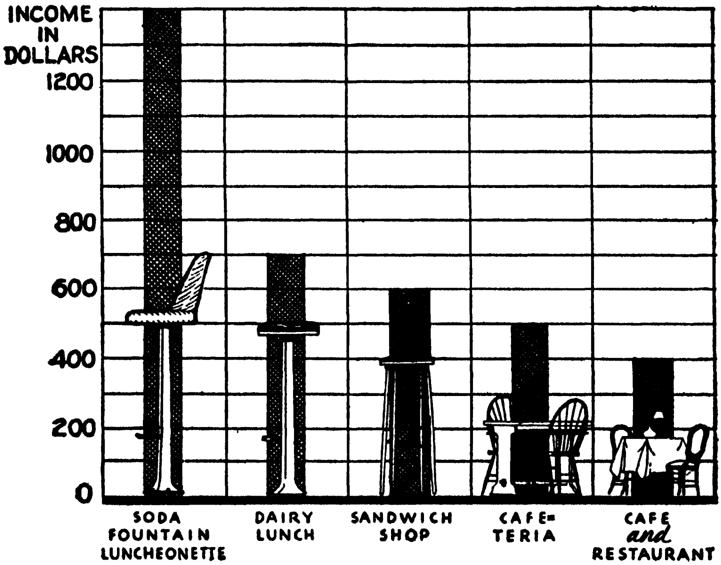


Source: United States Department of Labor

FIG. 74. NET IMMIGRATION TO THE UNITED STATES, 1929–1931 (VERTICAL BAR CHART)

Losses may be shown effectively by indicating them in red, although this method is generally confined to the non-printed diagram, due to the high cost of color printing. A more usual method is to use a different type of cross-hatching.

Pictorial Bar Charts. As previously indicated, the form of graph in which pictures are employed always has a greater attention value than any other type.



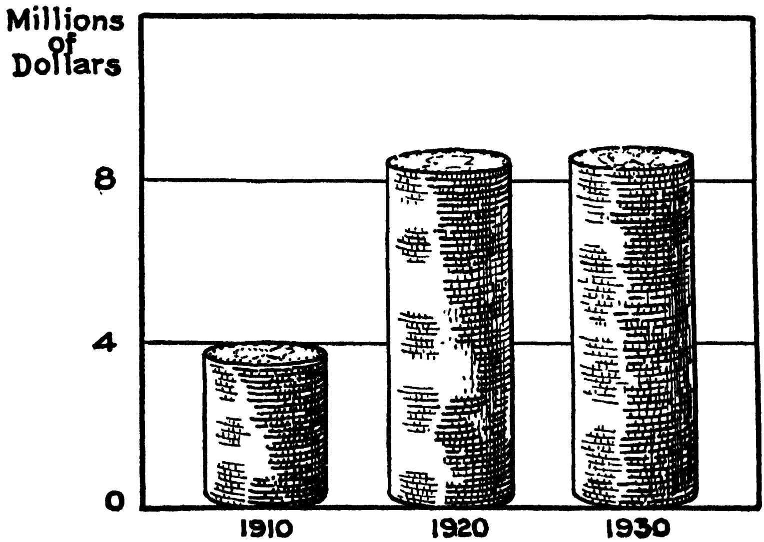
From an Advertisement in "Restaurant Management," October 1931

Fig. 75. Income Per Seat in Various Types of Restaurants (Pictorial Bar Chart)

In one form, the picture is simply superimposed on the bar chart. In Fig. 75 the income per seat in various types of restaurants is shown by the lengths of bars. There is superimposed on each bar a typical seat in the type of restaurant which the bar represents. The pictures in this instance serve not only to attract attention to the graph, but to aid in the identification of the bars as well.

The pictures of the objects dealt with may also be used as

bars and will then present a more striking contrast. The monetary stock of the United States for 1910, 1920, and 1930 are compared in Fig. 76 by means of three stacks of coins, the heights of which are proportional to the quantity of money at the three periods indicated.



Source: Annual Report of Secretary of the Treasury

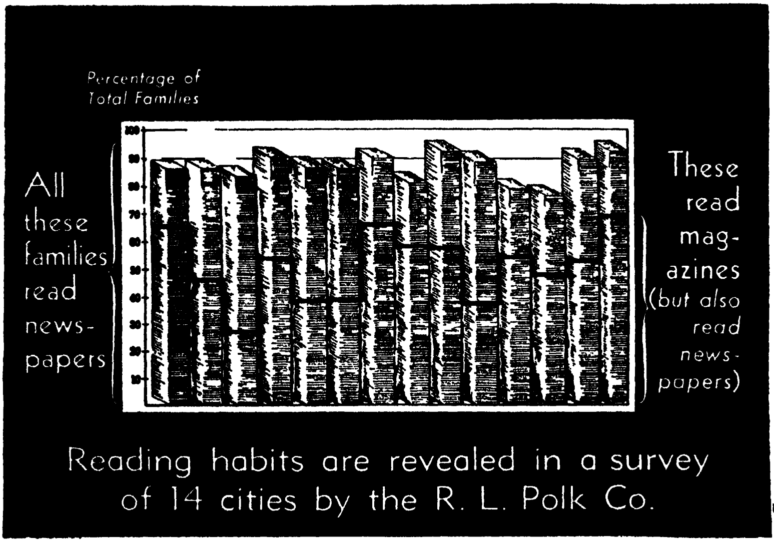
FIG. 76. MONETARY STOCK OF THE UNITED STATES, 1910, 1920, AND 1930 (PICTORIAL BAR CHART)

Fig. 77 indicates the reading habits of families in fourteen cities in a pictorial bar chart. The heavy black line was superimposed on the bars to indicate the percentage of families reading newspapers in addition to magazines.

The pictorial form of presentation is effective in the single-bar type of chart. It is essential that the object presented be fairly uniform in width throughout, so that *only* the lengths of the segments will vary.

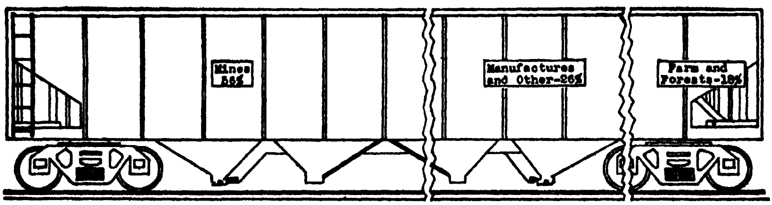
A diagram of this type, showing the distribution, by type of freight, of the freight tonnage of the Class I railroads in 1930 is seen in Fig. 78.

The Cosmograph. The Cosmograph is a mechanical device by means of which a special type of bar chart can be constructed. The device consists of a wood and bakelite backboard and a set



From The Newspapers, Bureau of Advertising, American Newspaper Publishers Association, 1934

FIG. 77. A PICTORIAL BAR CHART



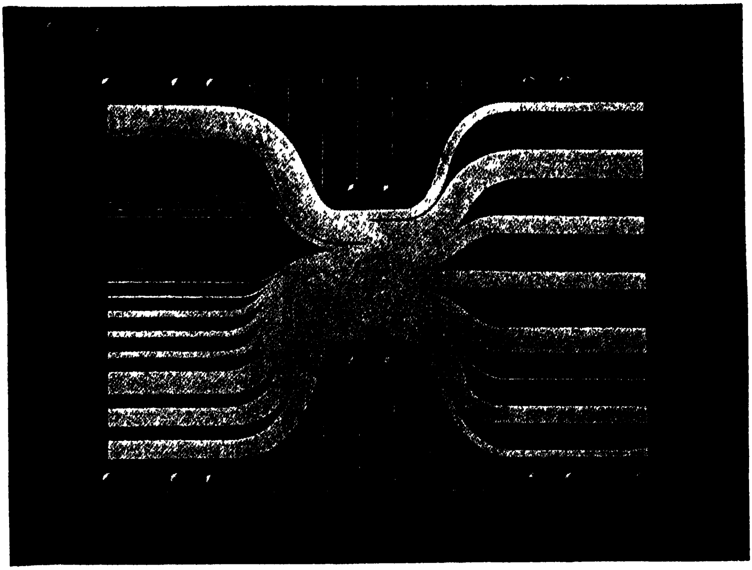
Source: Committee on Public Relations of the Eastern Railroads

FIG. 78. DISTRIBUTION OF FREIGHT TONNAGE ORIGINATING ON LINES OF CLASS I RAILROADS, 1930 (PICTORIAL SINGLE BAR CHART)

of 1,000 paper strips running horizontally across the board. Before the graph is prepared these strips are condensed into the form of a horizontal bar. The resulting graph is a 100-per-cent bar divided through its width rather than its length.

The cosmograph is especially useful in presenting counterbalancing items and their distribution. Income and disbursements, assets and liabilities, and other similar quantities, may be represented.

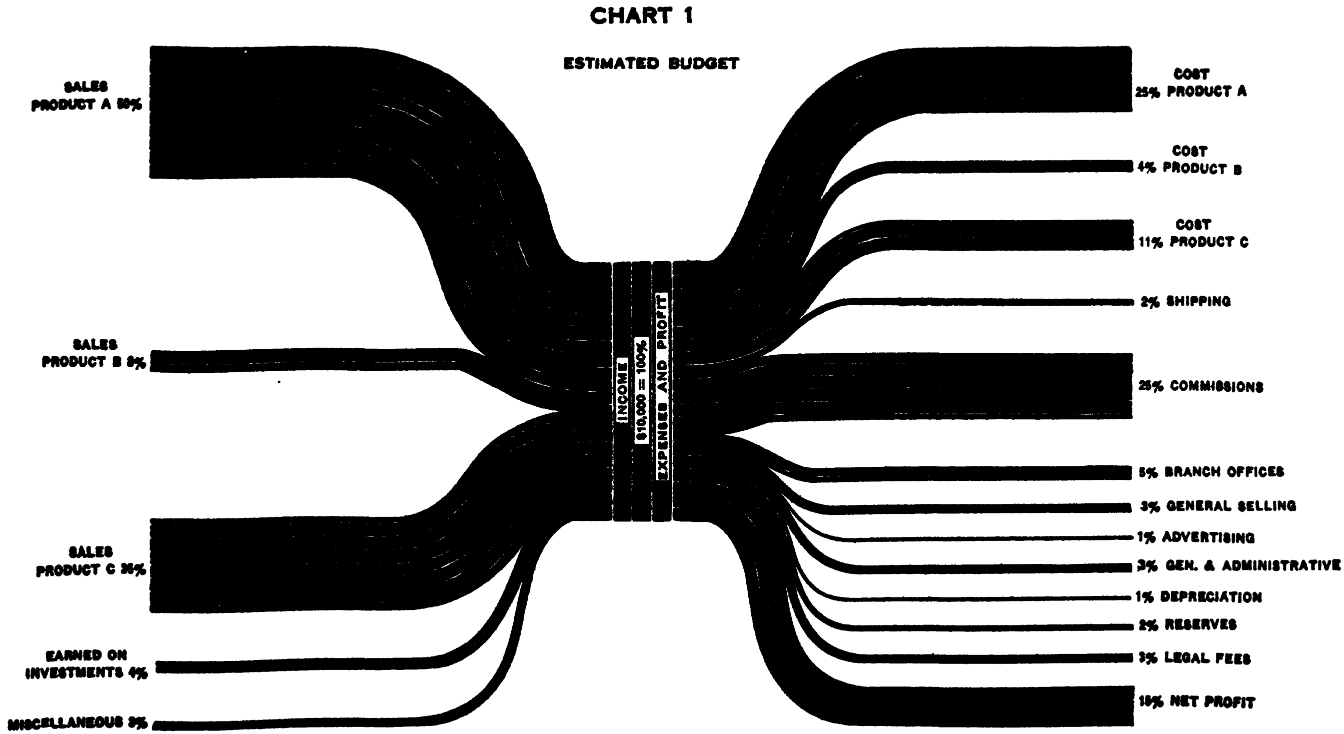
The width of the bar is used to represent 100 per cent. Since there are 1,000 strips of paper, each strip will represent 1/10 of 1 per cent. In Fig. 80, 50 per cent of the income is expected from



Courtesy of International Business Machines Co.
FIG. 79. THE COSMOGRAPH

the sales of product A. One-half of the strips (500) are grouped to represent this amount. Every tenth strip, representing 1 per cent, is a blue strip, and every fiftieth strip is a red one, indicating 5 per cent. The blue strips serve to facilitate the counting and grouping of the strips. The red strips reproduce as white and therefore serve to indicate the percentages involved. The white lines in Fig. 80, within the Product A group, are reproductions of these red strips.

When a group of strips is separated it is clamped together



Courtesy of International Business Machine, Inc.

FIG. 80. A GRAPH OF A BUDGET PREPARED ON A COSMOGRAPH

and held in position. A similar procedure can be followed for presenting the other items of income composing the total revenues. Appropriate numbers of strips are grouped together to indicate the remaining percentages. Expenditures are also arranged in percentage groups. The entire cosmograph is then photostated (negative only). Since a photostat negative reproduces black as white and white as black, the background will appear white and the strips of papers black (with the exception of the red strips, which will appear white). Captions may then be typewritten or printed on the photostat, or may be inserted before photostating.

The resulting graph is very effective in its appearance. The smooth curve of the bars afford a pleasing appearance. The two separate sections make it possible to present related data in one graph, a technique not available in any other form of graph. The unusual appearance of the graph tends to attract attention. In addition, a graph can be constructed quickly and easily with the aid of only the Cosmograph and a typewriter, without need of drafting or art work.

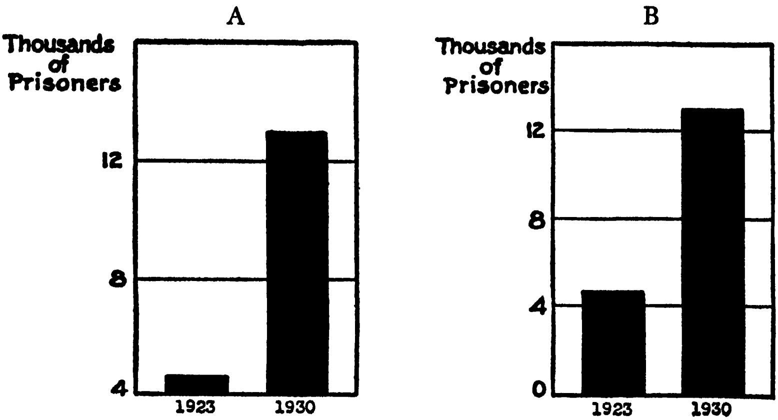
Rules to Avoid Common Errors Found in Bar Charts.

1. A zero line is always needed on bar charts. The scale break cannot be used in this type of graph, since the results would be decidedly misleading, due to the resulting change in the proportion between the length of the bars. The same data are presented in Fig. 81, showing the number of prisoners in Federal prisons and reformatories as of January 1, 1923 and 1930, in two different forms, in Graph *A*, with the zero lines omitted, and in Graph *B* in the proper fashion.

2. Only a limited number of background lines should be inserted on the grid. Since it is not intended that the actual values be read off the scale, but rather a rough visual comparison established, a great many fine divisions are unneces-

sary. Too many background lines tend to confuse the comparison.

3. The scale number should be placed *next* to the background line used to indicate that value, and not above or below the value. Improper placing of scale values makes it difficult to obtain even the desired rough estimate of the values of the bars.



Source: United States Bureau of Census

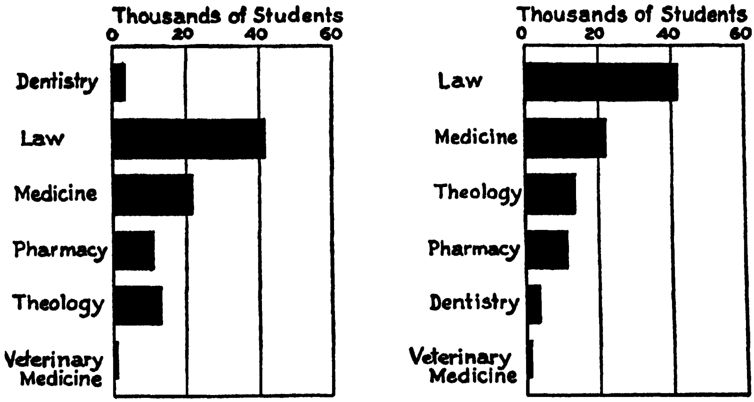
FIG. 81. NUMBER OF PRISONERS IN FEDERAL PRISONS AND REFORMATORIES AS OF JANUARY 1, 1923 AND 1930 (GRAPH A ZERO LINE NOT INCLUDED, INCORRECT FORM. GRAPH B ZERO LINE INCLUDED CORRECT FORM.) (VERTICAL BAR CHARTS)

4. As a general rule, the labels for the bars should not be placed within the bars since they tend to visually shorten the bars by the amount of space taken by the label. Labels should be placed immediately below the appropriate bars on a vertical bar chart, or to the left of the bar on a horizontal bar chart.

5. The bars should be arranged in a systematic order. They may be arranged according to time, with the earliest period first. If time is not involved, the bars may be ar-

ranged according to the sizes of the quantities involved, generally with the longest bar first, etc.

The advantage of this method over other forms of classifications will become obvious upon an examination of Graphs A and B in Fig. 82, which presents the number of students in professional schools in the United States by type of school,



Source: Office of Education, Department of Interior

FIG. 82. NUMBER OF STUDENTS IN PROFESSIONAL SCHOOLS IN THE UNITED STATES, BY PROFESSION, 1929-1930 (BARS PLACED IN ORDER OF SIZE)

1929-1930 first according to an alphabetical classification, and second according to magnitudes.

6. The segments in a bar should be arranged in order according to size.

7. In the component-part type of bar chart, when the segments are repeated in various bars, they should be retained in the original order, although the quantities may vary greatly.

8. As few segments as possible should be used in the component type of bar chart. Too many sections to a bar may serve to defeat its purpose by complicating the comparison.

CHAPTER VIII

AREA AND SOLID DIAGRAMS

Types of Area Diagrams—When to Use the Area Diagram—Area Diagrams—Regular Figures—Irregular Figures—Pictorial Area Diagram—Pie Diagram—How to Construct a Pie Diagram—Title, Source, Footnotes, Identification of Segments.
Types of Solid Diagrams—Construction of a Solid Diagram—Pictorial Solid Diagram—Regular Figures—Irregular Figures—Three-Dimensional Graphs—Rules to Avoid Common Errors in Constructing Area and Solid Diagrams.

QUANTITIES may be contrasted by the use of figures whose areas are proportional to the quantities they represent. An area diagram is thus a two-dimensional graph.

The area type of graph is less effective than the single dimensional graph, such as the bar chart, in conveying an accurate visual comparison. This may be seen in Fig. 83 in which the same quantities are contrasted by the two methods.

A ratio of 6 to 1 is represented in both graphs. In the bar chart the ratio is represented by the lengths of the bars that are compared. In the area diagram the ratio is represented by the areas of the figures.

Types of Area Diagrams. There are two types of area diagrams. In the first type areas of separate figures are contrasted to indicate the ratio between two amounts. In the second type, an area is subdivided in order to present a contrast between the component parts of a total.

Any type of geometric figure, or even irregular areas, may be used in an area diagram. However, it is more difficult to compare irregular figures.

In constructing the area form of graph it is advisable to use the simplest type of figure, such as the square or the circle,

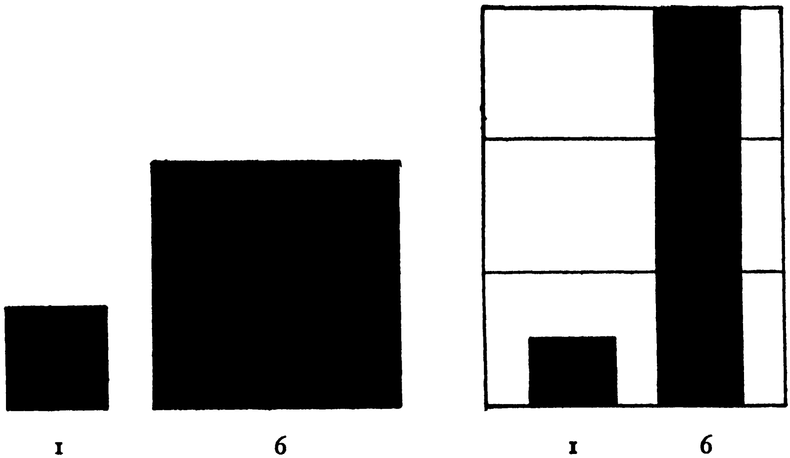


FIG. 83. AREA DIAGRAM AND BAR CHART SHOWING RATIO OF 6 TO 1

since comparison becomes more difficult as the type of figure grows more complex.

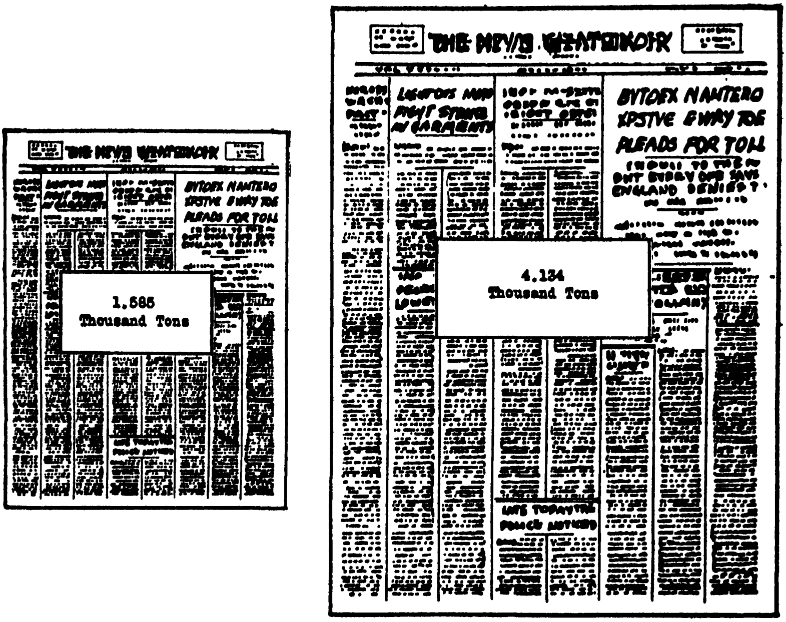
When to Use the Area Diagram. Although the area diagram is less effective than the single dimensional graph for presenting a correct visual contrast between quantities, it is a more unusual presentation, and thus attracts greater attention.

It should be used only when a few quantities are to be compared, generally not more than two or three. Usually the different amounts in an area diagram relate to places, kinds, types, or other qualitative specifications.

Area Diagrams. In the first type of area diagram, quantities are contrasted by comparing the total areas of several separate figures. These may be regular or irregular in form. It is difficult to make

a comparison of two dimensional figures, and therefore this type of diagram is less effective than the simpler type of bar chart.

In Fig. 84 the contrast between various quantities is presented in several forms by using geometric figures. The rule is that the



1921 1931

Source: United States Department of Commerce

Fig. 84. Imports of Newsprint Paper to the United States, 1921 and 1931 (Area Diagram)

ratio of the areas of the various figures must be the same as the ratio between the data given.

In Fig. 84 there is illustrated in the form of a pictorial area diagram a disparity in the newsprint-paper imports in 1921 as contrasted with 1931. The area of the newspapers depicted below represents a ratio of 1.0 to 2.6.

Irregular figures are frequently used for pictorial presentations. However, unless they approximate a regular figure in form their

areas are extremely difficult to determine.[1] The difficulty can be overcome, however, by establishing a given figure and then enlarging it in some fixed ratio by some such method as photostatic reproduction, etc.[2]

Pie Diagram. The pie chart is a circle, the area of which is divided into segments. Each segment represents a proportionate part of the whole. The resemblance to a pie cut into parts accounts for the name "pie" chart. This form of area diagram is of the subdivided type and is used to contrast the component parts of a single total. If more than one total is to be presented, several pie diagrams may be included on the same page.

How to Construct a Pie Diagram. In presenting data on a pie chart, relative proportions and not absolute data are depicted. Therefore, actual numerical data must be converted into percentage form for construction purposes.

1. Draw a circle. This may be prepared by means of a compass with a pencil or pen attachment.
2. Mark off the top center of the circle (90° on the protractor).
3. Let the circumference of the circle equal 100 per cent. From the center point on the circumference mark off that per cent of the total which the largest segment represents.

The segments in a pie chart should be marked off, in order, from the largest to the smallest. The size of each segment is obtained in the following manner:

The circumference of a circle is equal to 360°. Since the circle represents the total, or 100 per cent, it may be divided into 100 equal parts, with each part representing 1 per cent. In terms of degrees each per cent will then equal

$$\frac{360}{100} \text{ or } 3.6^\circ$$

[1] A map is a form of area diagram since, for example, sizes of political areas are compared on it.

[2] See Chapter XIV.

TABLE VI

TAX REVENUES OF THE UNITED STATES GOVERNMENT.
FISCAL YEAR 1932

Type of Tax	Income (In Millions of Dollars)	Per Cent of Tax Receipts
Income tax	$1,057	56.0%
Other internal revenue	504	26.7
Customs	328	17.3
Total	$1,889	100.0%

Source: Annual Report of the Secretary of the Treasury.

Thus in the table income tax receipts, the largest component, constitute 56.0 per cent of the total receipts, and therefore will be allotted 201.6° of the 360° of the circumference (56.0 per cent x 3.6°). Other internal revenue receipts, the second largest component, which constitute 26.7 per cent of the total, will be allotted 96.1° (26.7 per cent x 3.6°) and customs receipts the remaining 62.3° (17.3 per cent x 3.6°).

In order to better contrast the size of the segments each of the segments may then be filled in by cross-hatching, shadings, or colors.[3] Since it is customary to place the various segments in order of size, it is advisable to shade the cross-hatchings so that the darkest area will represent the largest segment, etc. (see page 153.)

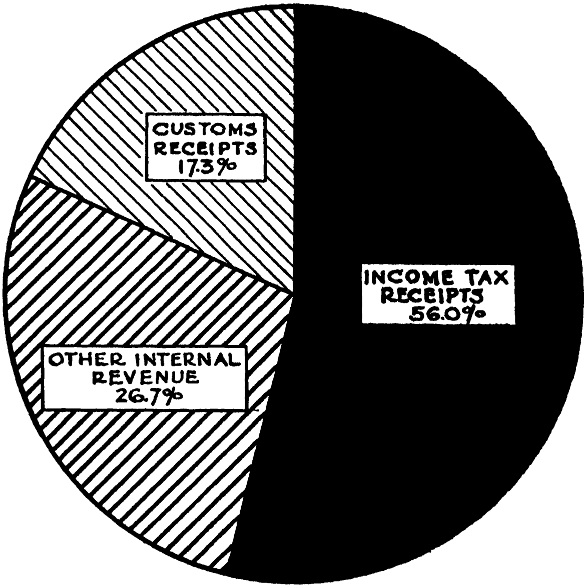
4. The title, source and footnotes may now be inserted in the same manner as indicated for other types of graphs.[4] immediately beneath each circle.

If more than one circle is used, the diagrams may be labeled

The various segments may be identified by means of a "window" inserted in each segment, containing the name and perhaps amount or per cent for that segment, as in the

[3] See page 153.
[4] See pages 16 and 18.

model pie chart shown below in Fig. 85, or by means of a key or legend placed in a conveniently located position beneath the graph, as in Fig. 61. Note the arrangement of the size of the segments and the use of windows in Figure 85,



Source: Annual Report of the Secretary of the Treasury
FIG. 85. TAX REVENUES OF THE UNITED STATES, FISCAL YEAR 1932 (PIE DIAGRAM)

showing the tax revenues of the United States, by type, for the fiscal year 1932.

Circles, squares, oblongs and other geometric figures may be split up to indicate the proportions of each part to the total. When the oblong is segmented in this manner, the resulting diagram is a single component part bar chart (see page 102).

The irregular figure is more frequently used for this purpose, especially in pictorial form where an object is split into parts so that the areas may be contrasted.

Solid Diagrams. Varying magnitudes may be compared by contrasting the size (volume) of solid figures, such as spheres, cubes, and other three-dimensional forms.

It is more difficult to make an accurate visual comparison with three-dimensional forms than with either the bar chart or the area diagram. Fig. 86 shows a comparison of quantities with a ratio of 4 to 1 in bar chart and in solid diagram form. The volumes of the forms must be compared, and therefore will have the same ratio as the original quantities (in this instance 4 to 1).

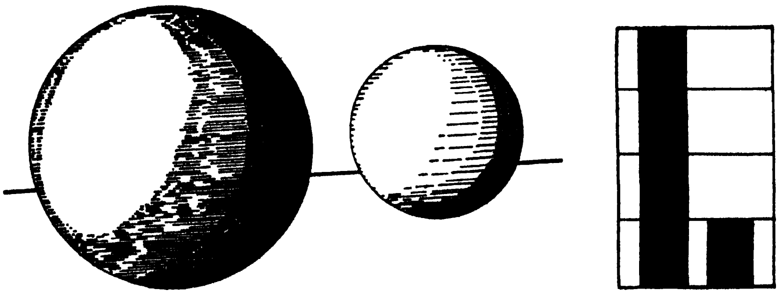


FIG. 86. DIAGRAMS SHOWING RATIO OF 4 TO 1 (SOLID DIAGRAM AND BAR CHART)

Types of Solid Diagrams. The solid or three dimensional graph may appear in various forms as a regular geometric figure, a pictorial graph, or three-dimensional line graph.

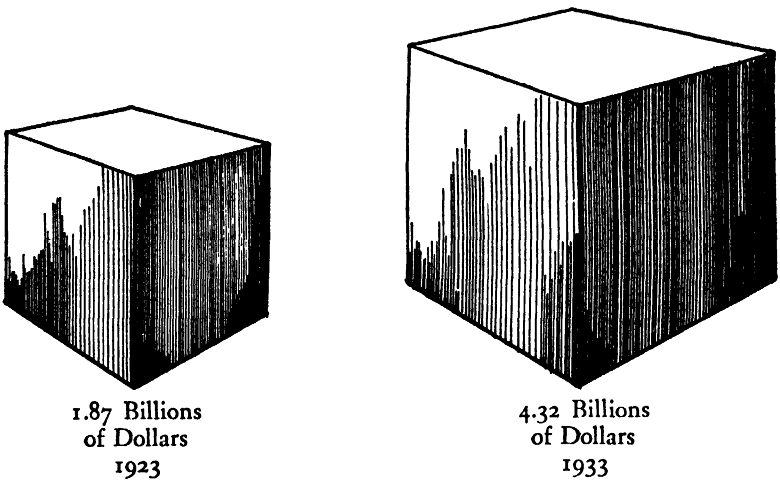
The type of diagram which makes use of regular forms is easiest to construct since its volume can be determined by formula.[5]

Construction of a Solid Diagram. To construct a solid diagram, a geometric figure of a suitable size is drawn *in perspective.* Its volume is computed by means of the correct formula. The volume of the contrasting figure is then determined by means of the ratio between the quantities to be compared. Thus if the ratio be-

[5] The volume of various regular figures can be computed as follows: Sphere, volume $= 4/3\pi r^3$; Cube-length x width x height; Cone $= 1/3\pi r^3$.

tween the quantities involved is 1 to 8, the volume of the second figure will be 8 times as great as the first.

The second figure, with a volume 8 times as great as the first may now be drawn *with all dimensions in proportion to those of the first.* If the volumes of the two figures are at a specific ratio (for example, 8 to 1) the ratios of their various proportions will be as the *cube root* of the ratios of the volumes. For two figures with a ratio 8 to 1 in volume, the ratios between the various pro-



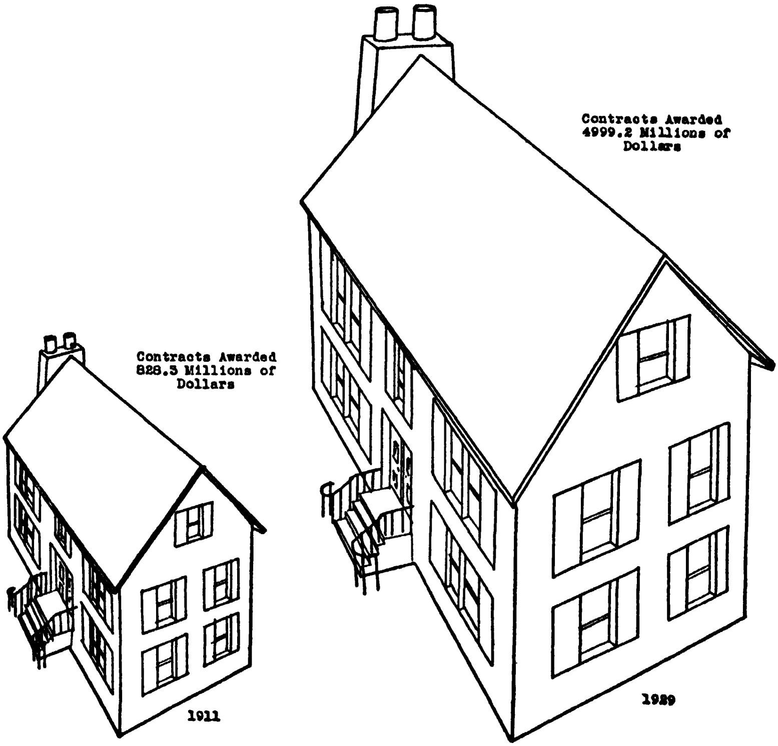
Source: Annual Report of the Secretary of the Treasury

FIG. 87. GOLD HOLDINGS OF THE UNITED STATES (SOLID DIAGRAM)

portions of the two figures (length, width, height, radius, etc.) will be as $\sqrt[3]{8}$ to $\sqrt[3]{1}$ or 2 to 1.

If two three-dimensional oblong figures with a ratio of 27 to 1 in volume are to be drawn and the smaller figure has a height of 2 inches, a length of 3 inches, and a width of 4 inches (volume $2'' \times 3'' \times 4'' = 24$ cubic inches), the dimensions of the other figure should be $\sqrt[3]{27}$ or 3 times as great. The dimensions of the second solid would be height 6 inches, length 9 inches, and width 12 inches. The resulting volume would be 648 cubic inches, or 27 times as great as the previous volume of 24.

The gold holdings of the United States for 1923, as compared with 1933, are shown by means of two cubes. The holdings in 1923 were $1.87 billions while the holdings for 1933 were $4.32 billions, or a ratio of 1 to 2.3. As in other solid diagrams it is important that the volumes of the figures be contrasted rather than the heights.



Source: F. W. Dodge Corporation

Fig. 88. Building Construction in the United States, 1911 and 1929 (Pictorial Solid Diagram)

Pictorial Solid Diagram. The most common form of solid diagram is the picture diagram. Building construction in 1911 as compared with 1929 is shown by means of two houses, the volumes of which have the same ratios as the amounts of construction or 1 to 5.9 in Fig. 88.

The production of oranges in 1921 and 1931 are compared in Fig. 89. In the 1921 season 44,319 carloads of oranges were produced; in 1931, 84,944. The ratio is 1 to 1.9.

The pictorial form of solid diagram is used frequently to portray comparative sizes of armies or navies or other similar relationships. Most people are misled by comparisons of this type, due to their failure to contrast the *volumes* of the figures rather than their heights or areas.

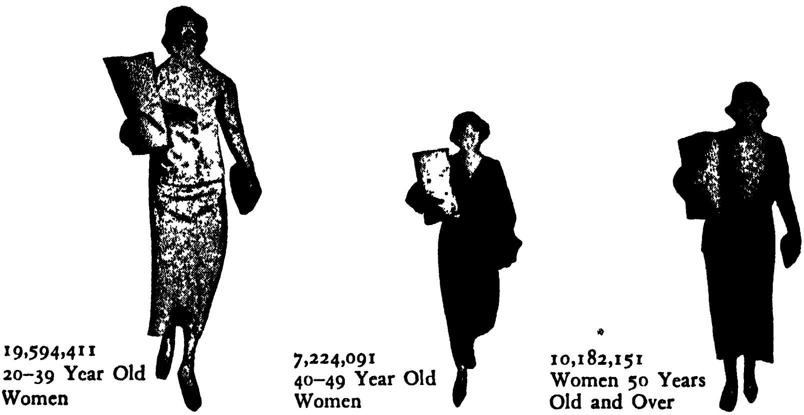
Source: Bureau of the Census, United States Department of Commerce

FIG. 89. PRODUCTION OF ORANGES IN THE UNITED STATES, 1921 AND 1931 (PICTORIAL SOLID DIAGRAM)

In the pictorial solid diagram shown below it is necessary to make a comparison of irregular figures (the sizes of women in the graph) in order to determine the distribution of women of various age groups in the United States in 1930.

The variation in the volume of figures used in solid pictorial diagrams can be made much more effective if an effort is made to use figures which truly portray not only the relative or proportionate volumes of data, but also the meaning of the data. Thus, in this figure, not only did the volumes of the figures used make a comparison of the quantities involved, but the figures themselves were representative of the data they depicted, in this case women of different ages. The pictorial graph is striking and therefore its attention value is greater than that of any other

type. In addition, the novelty of this form of presentation tends to cause the message to be retained for a much longer period of time. The pictorial form of graph provides an opportunity for introducing the element of variety to eliminate monotony in the presentation of many graphs. Some of these advantages may be obtained by making use of the line graph, bar chart, or area diagram, with a picture or series of pictures contained within the graph. A graph of this type is shown on pages 111 and 120.



Source: United States Bureau of the Census

FIG. 90. AGES OF WOMEN IN THE UNITED STATES, 1930
(PICTORIAL SOLID DIAGRAM)

Three-dimensional Graphs. Since three dimensional graphs make use of a three dimensional grid, they are used to present the relationship between three series of data. This subject will be discussed in Chapter VIII which deals with the graphing of related series.

Rules to Avoid Common Errors in Constructing Area and Solid Diagrams.

1. The proportion between the *areas* of the figures in an area diagram must be in the same ratio as the numerical data. The heights of the pictorial figures should not be used to indicate a ratio for an area diagram. If the lengths of the

figures have been compared, a bar chart, and not an area diagram, has been constructed.

2. The segments in a pie diagram should be constructed in order of size. If two or more pie diagrams are compared, the same components in each separate pie diagram should have the same color or shade.
3. There should be a minimum number of components on a pie diagram. If possible, small segments, which are relatively unimportant, should be grouped into a single segment.
4. The *volume* of figures in a solid diagram, and not the heights or lengths of the figures, is compared. The use of the heights of a bar greatly exaggerates the comparison of the figures. The heights should be in a proportion of the cube roots of the volume of the figures compared.
5. Simple regular figures, as squares, circles, etc., should be used in presenting solid and area diagrams. Irregular figures are difficult to construct and inaccuracies are liable to result.

CHAPTER IX

GRAPHING RELATIONSHIPS

The Graph of Relationship—How to Construct a Scatter Diagram—The Law of the Relationship—Scatter Diagram—The Grid—The Scale—Plotting the Points—Title—Source—Three Dimensional Relationship Graphs—Isometric Ruling—How to Construct a Model Three-dimensional Graph.

IT IS frequently desirable to present in graphic form the relationship between two variables or factors, such as the association between the price of crude oil and of gasoline, the strength of a cable of a certain type and its gauge, the amount of an electrical current passed through a solution and the amount of a substance deposited by the electro-chemical reaction, the intelligence of a group of students (as measured by their intelligence quotients), and their grades on an examination, etc..

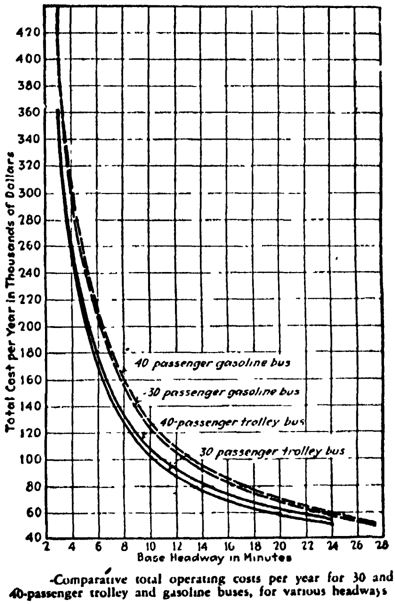
The Graph of Relationship. Associated data always consist of a pair of series with a pair of values for each identified value, such as period of time, place, or other identifying item.

A grid of the type used in preparing line graphs may be constructed, and a scale of values for one of the variables prepared, on the horizontal (X axis), while an appropriate scale may be established on the vertical or Y axis for the other series. If such a pair of values (for each year, state, person, etc.) is then used to locate a point, the resulting graph indicates the relationship between the series.

If the relationship is perfect—that is, if there are no other

factors which tend to cause the values to depart from this "law"—there can be only one Y value for a given X value, and the points will describe a curve or straight line across the face of the graph. If the points follow such a curve and are connected, the resulting curve may be said to describe the *"law" of the relationship.*

In the figure below there is depicted the relationship between



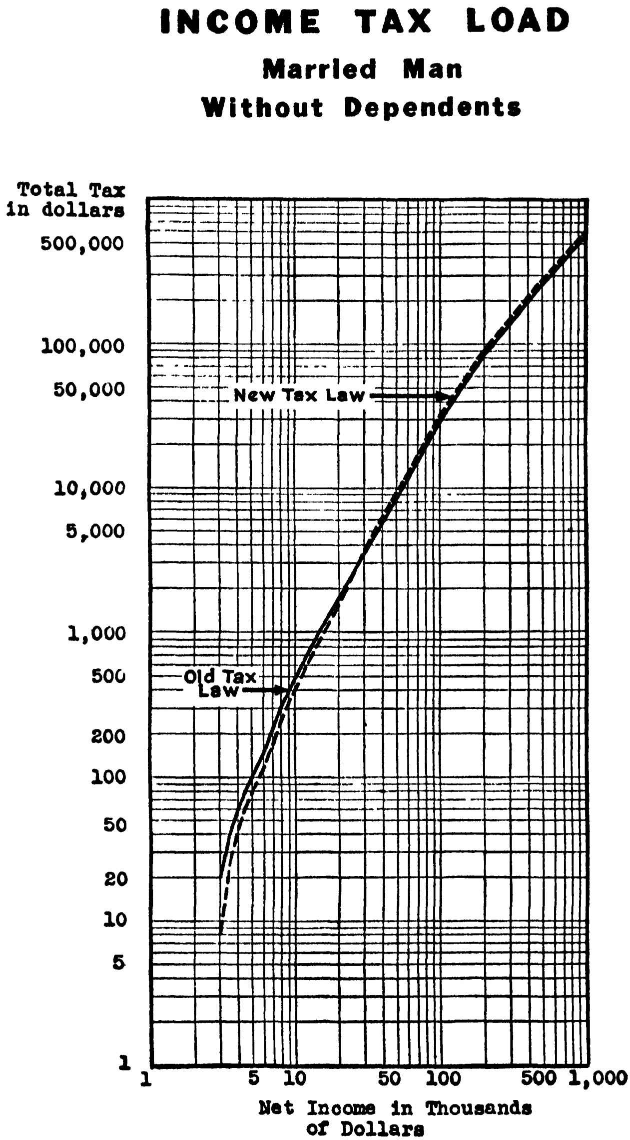
From "Electrical Railway Journal" September 1931

FIG. 91. A DIAGRAM OF RELATIONSHIP

headways and total costs of operation for various types of gasoline and trolley buses.

Diagrams of relationship may also be prepared on grids with other than arithmetic rulings. Semi-logarithmic, logarithmic, and other types of grids (see Chapter VI) may be used. A diagram of relationship on logarithmic paper is shown in Fig. 92.

In this diagram the relationship is shown between personal net and total income tax on net income under two different income-tax laws.

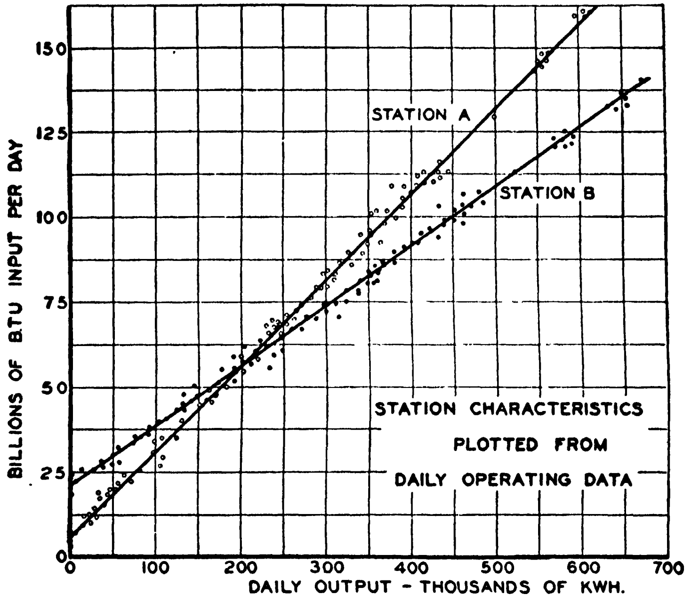


From "Business Week," May 12, 1934

FIG. 92. DIAGRAM OF RELATIONSHIP ON A LOGARITHMIC GRID

If the relationship between the two series is very poor or non-existent, the points will be scattered at random across the face of the graph. A graph of this type is known as a *scatter diagram.*

Usually the points tend to cluster together in the form of a path which follows a definite movement across the face of the



Average characteristics for two typical stations as obtained from daily operating data

From "Electrical Engineering," September 1931

Fig. 93. A Scatter Diagram

scatter diagram. The higher the degree of association, the more compact the area within which the scatter of the points will be limited.

The general movement of the "path" across the grid is descriptive of the type and form of relationship. A line may be drawn by free-hand method or fitted by mathematical technique to best describe the trend or direction of this movement. The scatter diagram in Fig. 93 shows the relationship between British Thermal

Unit input and the daily output in kilowatt hours for two different stations as determined from daily operating data.

The degree of scatter may then be used as an approximation of the degree and form of relationship between the two series, although more technical methods must be used if a careful determination is necessary. The most commonly used method of determining this relationship is the least-squares method.[1]

How to Construct a Scatter Diagram.

1. Prepare a grid or background ruling. This grid is of exactly the same type as that used for line graphs. The method of constructing a grid of this type and the rules to be observed are completely outlined on page 47.
2. Select and arrange a suitable scale of values on both axes of the grid. The scale of values for the independent (or causal) variable is placed on the horizontal (X) axis, and the scale for the dependent variable is indicated along the vertical (or Y) axis. As explained in the chapter on line graphs, the scale is determined in the following manner:
   a. Establish the range (the limits within which the values fall and the number of scale divisions desired).
   b. The range is then divided by the number of divisions.

   The range of magazine circulation in Table VII is from 21,611 to 1,618,244 or about 1,596,633. Dividing the range by the number of scale divisions desired, 4, the result will be 399,158. The scale divisions may be rounded off to 400,000 and this value used as the interval for the scale on the X axis. The same procedure may be followed in preparing the scale for the Y axis.

   It is also necessary to have two *scale captions,* identifying the two scales and indicating the two units used. The

[1] See Arkin, Herbert, and Colton, Raymond R., *Statistical Methods,* 4th ed.; Chapter IX, New York, Barnes and Noble, 1939.

TABLE VII

CIRCULATION AND ADVERTISING RATE PER PAGE FOR SELECTED MAGAZINES IN THE UNITED STATES IN 1932

Magazine	*Circulation*	*Rate per Page (In Dollars)*
A	561,347	1,100
B	661,150	1,837
C	324,941	1,700
D	290,624	1,500
E	1,902,246	4,500
F	994,273	3,100
G	401,477	1,350
H	28,808	450
I	41,787	300
J	1,416,843	4,050
K	1,250,403	3,247
L	102,236	125
M	187,754	1,190
N	21,611	60
O	138,492	1,070
P	81,716	220
Q	586,205	1,600
R	1,618,244	4,200
S	164,098	200
T	1,370,927	3,000
U	288,846	1,200
V	134,766	800
W	117,397	1,000

Source: Standard Rate and Data Service, 1932

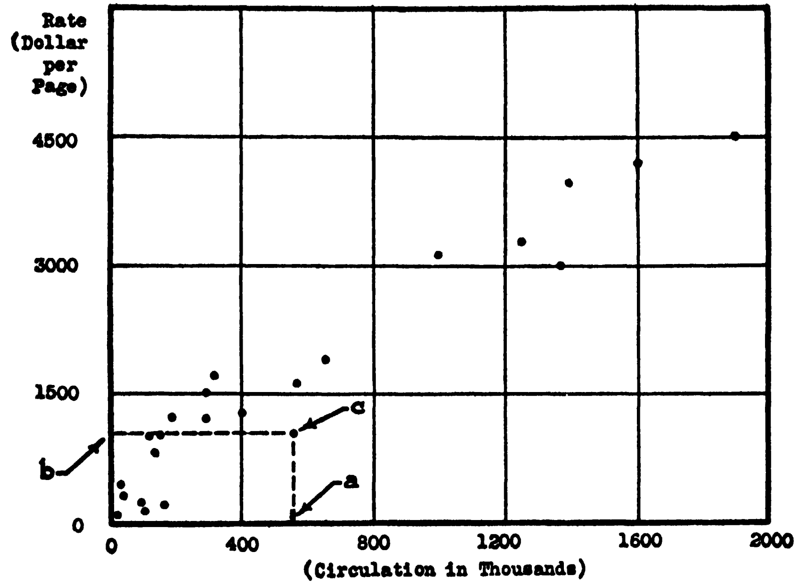
scale caption for the X axis is placed just below that scale at the center, while the scale caption for the Y axis is placed at the top of the scale.[2]

3. Plot the points. The first pair of values, 561,347 for circulation and a rate of $1,100 for magazine A (see Table VII) may now be used to locate a point on the grid. By reference to the scale for circulation, the given value of 561,347 is located (point *a*), while the corresponding value of $1,100 is located on the scale (point *b*) on the Y axis. A dot may now be made at the position indicated by the scale values (point *c*). This is continued until all of the pairs of values have been used to locate points.

A *title* is placed directly above the grid. This title must

[2] See page 55.

be complete and self-explanatory and must conform to the previously outlined rules for graph titles. The *source* of the data is placed immediately beneath the grid at the lower left-hand side.



Source: Statistical Rate and Data Service, 1932

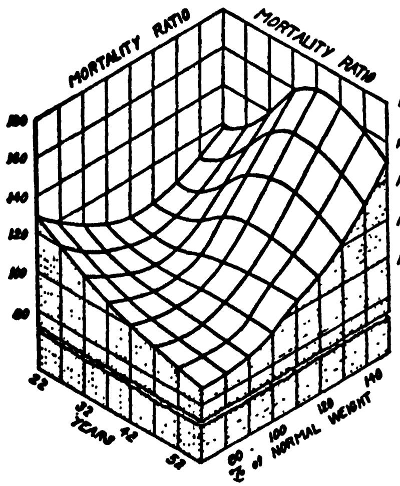
FIG. 94. CIRCULATION AND ADVERTISING RATE PER PAGE FOR SELECTED MAGAZINES IN THE UNITED STATES IN 1932 (SCATTER DIAGRAM)

Three-dimensional Relationship Graphs. The relationship diagrams outlined above provide for indicating the effect of, or association with, only one predominant factor. When there is more than one predominating variable, resort must be had to three-dimensional graphs.

A relationship which is commonly discussed, but rarely depicted, is the relationship between weight, age, and mortality. The mortality rate, is dependent upon not only the age of a person, but also upon his weight. This relationship may be shown graphically as in Fig. 95.

To determine an individual point on the graph it is necessary to have reference to three scales. By erecting perpendiculars for each scale at the appropriate position the points to be plotted will be located.

Three-dimensional graphs can be constructed either as models actually using all three dimensions, or on the axonometric charts



Reproduced from Measuring Joint Causation, Court, A. T., "Journal of the American Statistical Association," September 1930

FIG. 95. RELATION BETWEEN WEIGHT, AGE, AND MORTALITY (THREE DIMENSIONAL GRAPH)

which make use of three axes drawn on a sheet of paper (actually only two dimensions).

The three-dimensional graph has been constructed in many forms. Wood, paper, plaster-of-paris, or any plastic material can be used for such a purpose.

When the three-dimensional graph is constructed in the form of an axonometric chart on a two-dimensional surface (sheet of paper), it may be constructed either in perspective or without regard to perspective. When drawn to perspective it is a sketch of a model. Drawing the graph to perspective causes difficulty

in the arrangement of the scale, since the scale values can no longer be equally spaced.

If perspective is disregarded, the graph may be prepared by erecting a vertical axis and drawing two other axes at angles of 60° to this first axis. This type of grid is known as an *isometric*

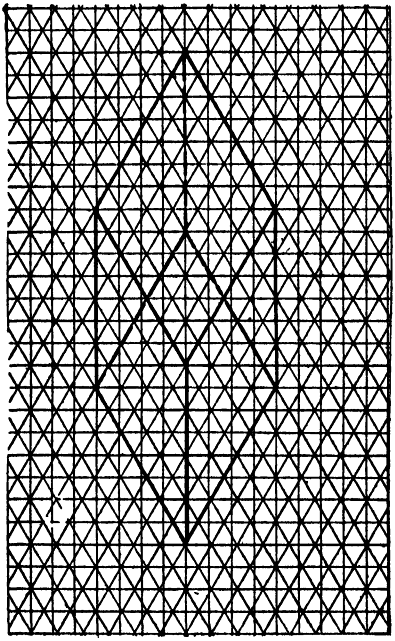


FIG. 96. A CUBE DRAWN ON ISOMETRIC PAPER

ruling. Printed sheets with isometric rulings can be readily purchased.

Isometric rulings are generally useful in preparing sketches of solid objects. Fig. 96 shows an isometric printed background sheet with a cube drawn on it. It is only necessary to follow the lines of the background ruling to prepare such a sketch, but the resulting diagram does *not* have perspective.

How to Construct a Three-dimensional Graph Model. The three-dimensional graph model has a tremendous advantage in

that it can be turned and examined from any angle, while the drawn chart can be seen from one side only.

The model graph can be readily constructed out of cardboard if the form of the relationship between each independent and the dependent variable does not change with changes in the other variable. On the three-dimensional graph there are three variables, one placed on the vertical axis (Y axis), and two placed on the two horizontal (X and Z) axes. If the finished model graph is looked at from any one side, the relationship between only two of the variables can be seen (X and Y or Z and Y). Since the grid is in the form of a cube, the four sides of the cube may be constructed first.

The first step is to graph the relationship between the X variable and the Y variable on an arithmetic grid drawn on cardboard. Two graphs of this type are prepared, since two of the sides are alike. The cardboard is then cut along the plotted curve line and fastened in a vertical position to a baseboard. The same process is followed for the relationship between the Z and the Y variable, and these sides are also placed to form the base of a cube.

Another piece of cardboard is bent into position and fastened as a cover for the four sides.

This type of model can be used when the form of relationship does *not* change with changes in the size of the other independent variable—that is, if the relation between X and Y is of the same type (say a straight line) for all values of Z. If the relationship is a changing one it will be necessary to erect a series of sides parallel to the original ones and placed within them, showing the relationship occurring at various values of the other independent variable.

CHAPTER X

MAP GRAPHS

Types of Map Graphs—Construction of the Map Graph—Outline Map—Shaded Map—Cross-hatched Map—Cross-hatchings—Colored Maps—Dot Map—Pictorial Dot Map—Traffic Maps—Pin Maps.

THE map graph performs the function of other types of graphs in comparing quantities, and in addition indicates the geographical location of these quantities. An example of a map graph is shown in Fig. 97.

Types of Map Graphs. Map graphs may be classified in accordance with the method used to contrast and locate the given quantities, as follows:

1. Shaded or cross-hatched maps
2. Colored maps
3. Dot maps
4. Pin maps

Construction of the Map Graph.

1. Draw or obtain an outline map of a suitable size. These maps may be purchased in a variety of sizes and for entire countries or sections of a country. The best type of outline map for this purpose is one with the smallest possible amount of detail. Only the borders of the country or section used should be indicated, with perhaps an indication of the boundaries of the more important political areas, such as states, etc., and the more important

rivers and lakes. An outline map of this type is shown in Fig. 98.

2. Locate the given quantities and decide upon a method of contrasting them. As indicated above, a number of methods may be resorted to, including shading, cross-hatching, colors, dots, pins, etc.

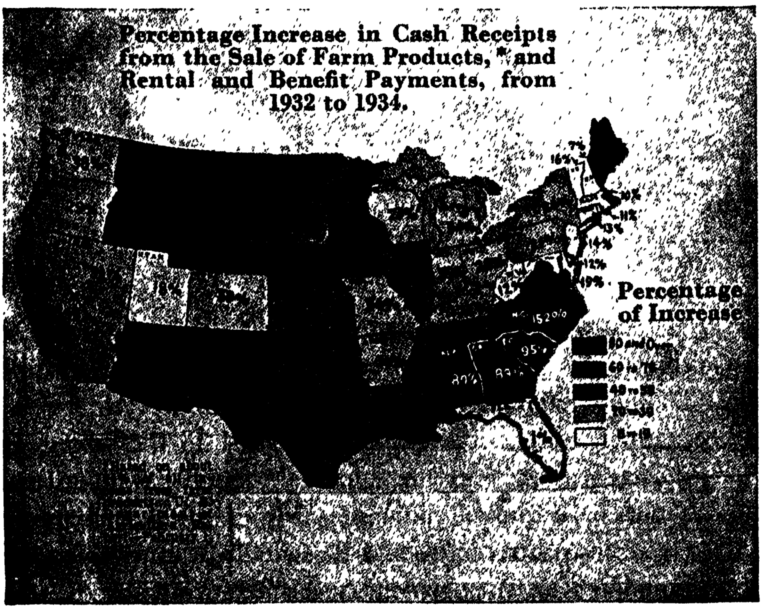
From "Nation's Business," May 1931

FIG. 97. BUSINESS CONDITIONS IN THE UNITED STATES AS OF APRIL 1, 1931 (MAP GRAPH)

Relative quantities for each area may be indicated in shaded maps through variations in the degree of shading. Shading ranges from solid white to solid black. In this manner, magnitudes from the highest to the lowest are depicted as seen in the key of figure below. The map shows by means of different shadings the percentage increase in cash receipts from the sale of farm products and rental and benefit payments to farmers from 1932 to 1934.

In making use of shading, various selected groups, ranging from the lowest to the highest, are determined, and assigned corresponding degrees of shading, ranging from

FIG. 98. AN OUTLINE MAP



From an Advertisement in "Printers' Ink Weekly," February 21, 1935

FIG. 99. A SHADED MAP GRAPH

white to solid black. White may be used to indicate the absence of the item studied. In Fig. 99 the largest quantity group (80 per cent and over) was assigned solid black, and the other groups varying shades down to white, which indicates no increase.

The objection to shading is that fixed gradations in shading are difficult to produce. The Ben Day[1] process may be used for this purpose. An additional disadvantage results from the difficulty of reproduction, in that a half-tone must be made rather than a line cut, a more expensive process.[2]

Cross-hatched maps indicate differences in quantities by the use of diagonal lines and variation in the proportion of black

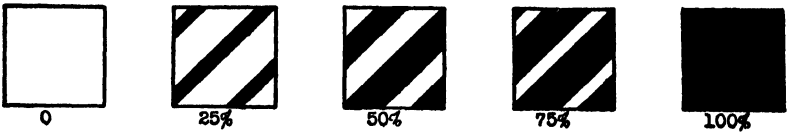


FIG. 100. CROSS HATCHINGS

and white space used. The cross-hatchings start with white for either the smallest quantity, or as an indication of absence of this item (zero), and the next quantity group is indicated by light diagonal lines. The thickness of the diagonal lines is then increased as the quantity increases, thereby reducing the amount of white space shown in the area until it is solid black (no white space at all).

A more exact visual comparison is made possible through the use of this type of map graph in that the largest quantity may be established as 100 per cent, and the other quantity groups may be indicated by using cross-hatchings in which the proportion of black space may be varied so that it will correspond to the per cent that the given group is to the total. Cross-hatchings with varying percentages of black space are shown in Fig. 100.

[1] See page 220. [2] See page 220.

An example of a cross-hatched map graph is shown in Fig. 101.

Although *colored maps* are striking and therefore have greater attention-attraction value, they have not as much value in making the comparison between the given quantities. Since there is no progression in the colors, variations in magnitude cannot be conveyed in the same manner as the shaded or cross-hatched map. Blue does not indicate a greater or smaller quantity than orange.

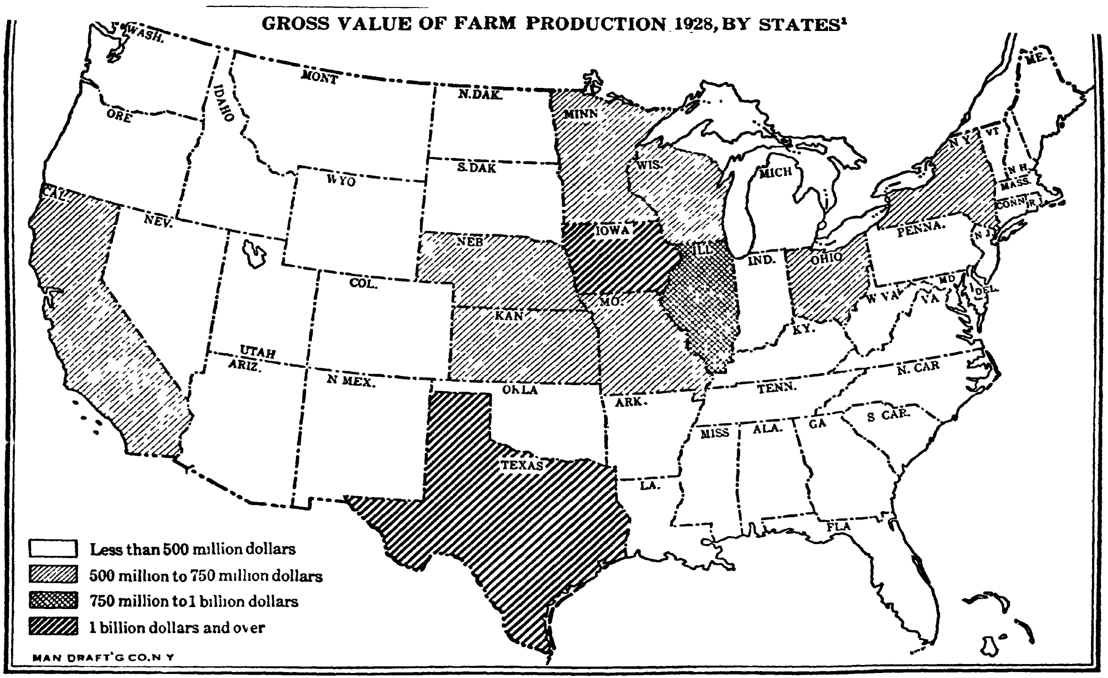
To overcome the difficulty of lack of progression, various shades of a given color may be used. This procedure is subject to the limitation that there are comparatively few distinct shades of a particular color available. If this technique is resorted to, the resulting graph is not a true color map graph, but rather a shaded map graph in color.

In preparing a colored map graph, only a small number of colors should be used or the graph will be difficult to interpret. An additional difficulty arises in the reproduction of colored graphs. Color cannot be reproduced in the most commonly used processes of reproduction. The color reproduction processes are often prohibitively expensive.

The *dot-map graph* which indicates quantities by the use of dots, is one of the most frequently used graphs. Variations in quantities may be indicated by varying the size of the dots employed, or by varying the number of dots used without changing their size. The dot used need not be round, but may have any geometric form.

If the size of the dot is varied, the result is an area diagram superimposed on a map. The ratios between the *areas* of the dots must be the same as the ratios between the quantities involved. Dots of varied size are used frequently to identify quantity groups rather than to attempt to vary the size of the dot for each individual value.

The *dot-map graph* may be constructed by using dots of a fixed size to denote quantities by assigning to each dot a given value and varying the number of dots used. Thus, in Fig. 103 each

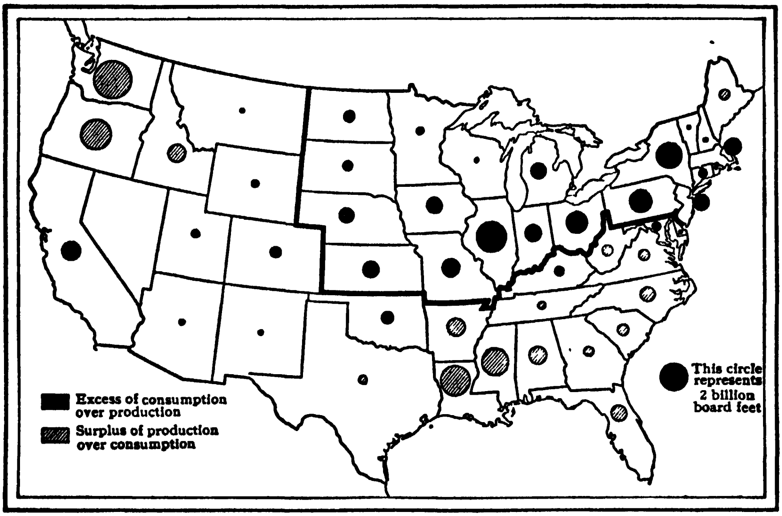


[1] *Yearbook of Agriculture*, 1930, 974-975.

From Faulkner, H. U., "American Economic History," Harper and Brothers, 1935

FIG. 101. GROSS VALUE OF FARM PRODUCTION BY STATES IN 1928
(CROSS HATCHED MAP GRAPH)

dot represents $5,000,000 and the annual value of mineral production in South America is indicated by placing the appropriate number of dots in the area considered. A similar procedure was followed in Fig. 104, showing the distribution of wheat production in sections of China. If this type of graph is to be used, care must be taken in the selection of the size of the dot and the quan-



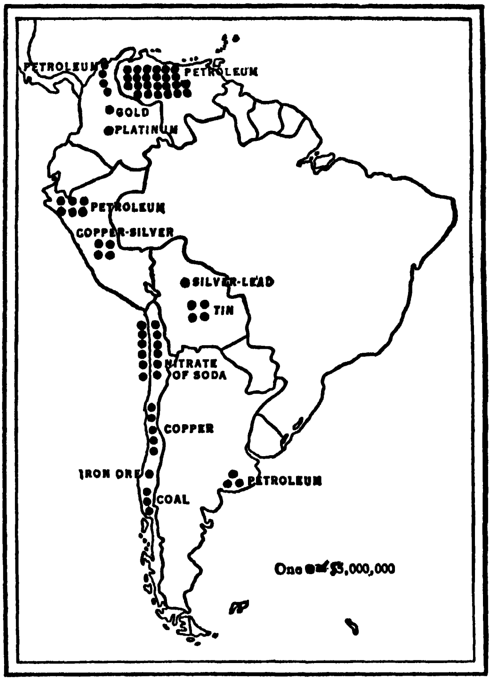
From Whitbeck, Ray Hughes, "Industrial Geography," p. 40

FIG. 102. TIMBER SHORTAGE OR SURPLUS FOR ONE YEAR BY STATES FOR THE UNITED STATES (DOT MAP GRAPH)

tity it is to represent. If the dot is too large or the quantity it represents is too small, there will be too many dots in the area and they will overlap.

If a large quantity appears in a limited area, when very small dots are used to represent small quantities, the great density of dots gives the impression of a large quantity. However, when large dots are used to represent large quantities, they may be arranged systematically so that the quantity indicated for a particular section may be seen at a glance. The use of each type of dot on a map graph may be seen in Figs. 103 and 104.

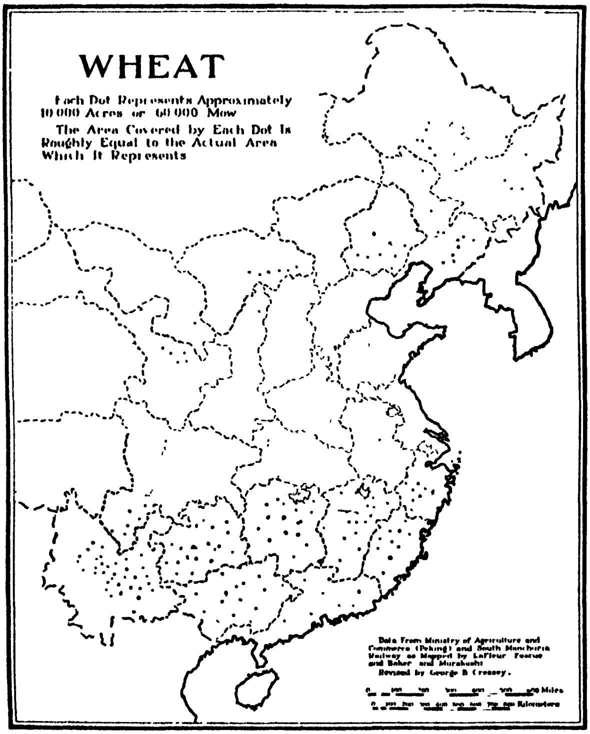
The construction of the dot map graph is difficult, due to the technique involved in drawing small dots. For this purpose the drop-curve pen must be used if they are to be constructed neatly. An additional difficulty arises when the size of the dot is varied because the areas of small-sized dots must be computed. In constructing such circular dots care should be taken to draw the dots on the basis of proportional areas rather than proportional diameters.



From Whitbeck, Ray Hughes, "Industrial Geography," p. 311
FIG. 103. APPROXIMATE ANNUAL VALUE OF THE CHIEF MINERAL PRODUCTS OF SOUTH AMERICA (DOT MAP GRAPH)

Pictorial Dot Map. A much more striking dot map can be drawn by using small pictures rather than geometric figures for the dots, especially if these pictures are related to the type of data.

In Fig. 105 the population of each of the countries of the Far East is represented by means of a series of pictures of men. Each man on the map represents 20 million of population. Considerable in-



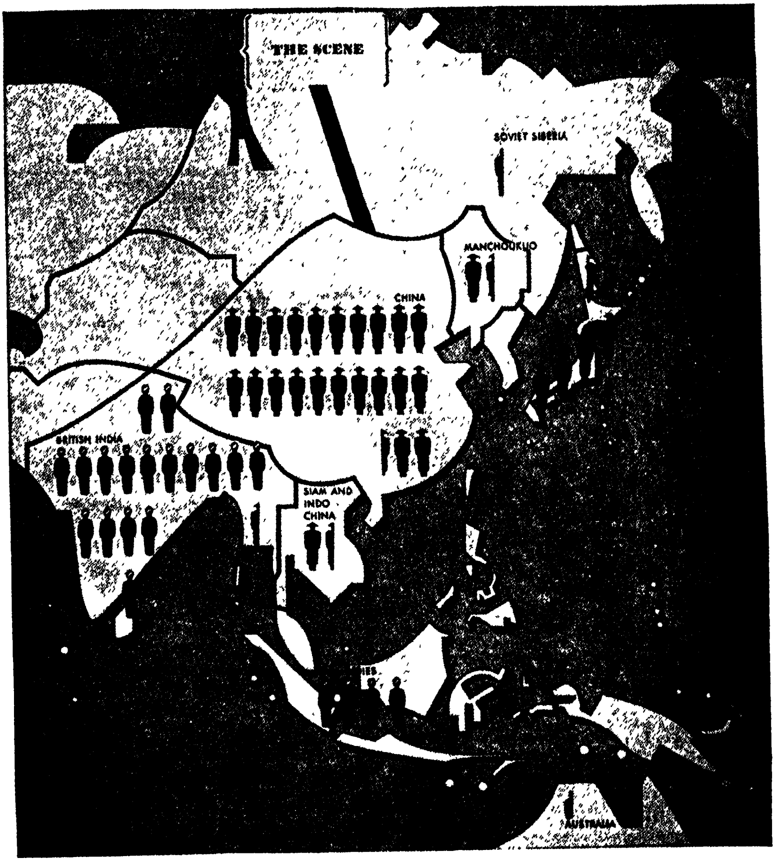
Reproduced from Agricultural Regions of Asia, Cressy, G. B., "Economic Geography," April 1934

FIG. 104. DISTRIBUTION OF WHEAT PRODUCTION IN THE NORTH AND YANGTZE VALLEY OF CHINA (DOT MAP GRAPH)

terest was added by using figures resembling in costume the population of the various countries.

Traffic Maps. The flow of traffic and its density may be indicated on a highway map by widening the line allotted to the highway on the map in proportion to the density of traffic on the road.

The density of traffic on the highways surrounding New York City is shown in Fig. 106. Maps of this type are widely used as



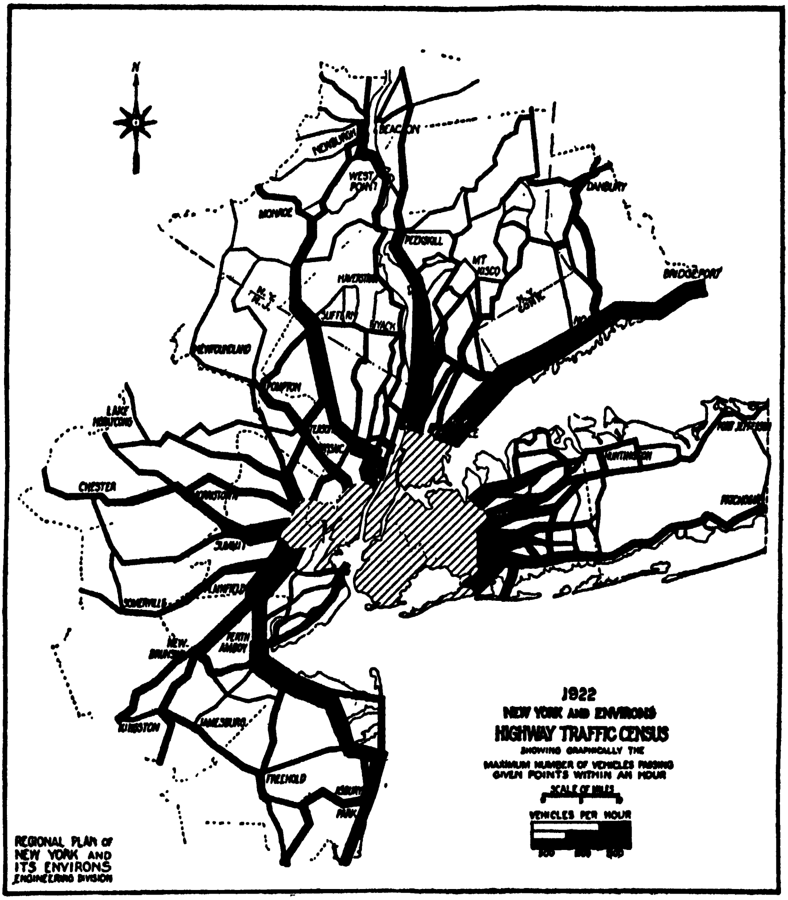
From "Asia," February 1935, Courtesy Pictorial Statistics Inc.
Fig. 105. A Pictorial Dot Map

a basis for planning highway improvements and relieving traffic congestion.

Pin Maps. In recent years there has been a growing use of pins, tacks, flags, etc., on maps to denote specific statistical data.

An important use of this type of graph is in sociological work.

In a large Eastern city the police department has been able, by using various-sized and different-colored pins, to record graphically the number, type, and cause of accidents occurring on each

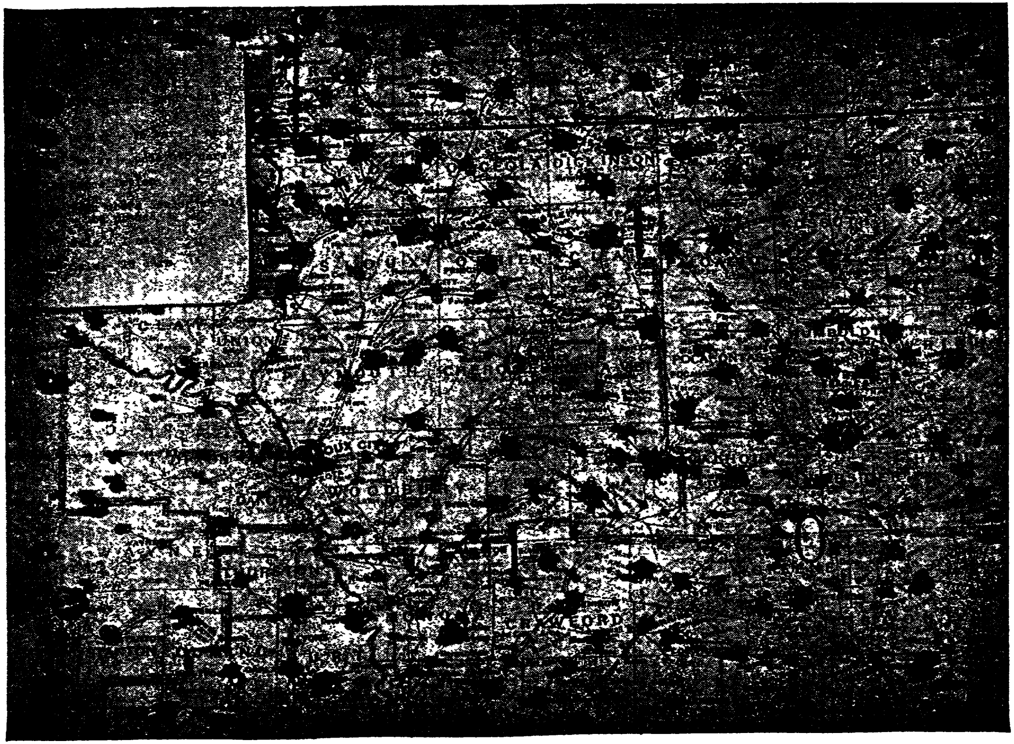


From "Regional Survey of New York and Its Environs," Lewis, H. M. and Goodrich, E. P.

FIG. 106. A TRAFFIC MAP

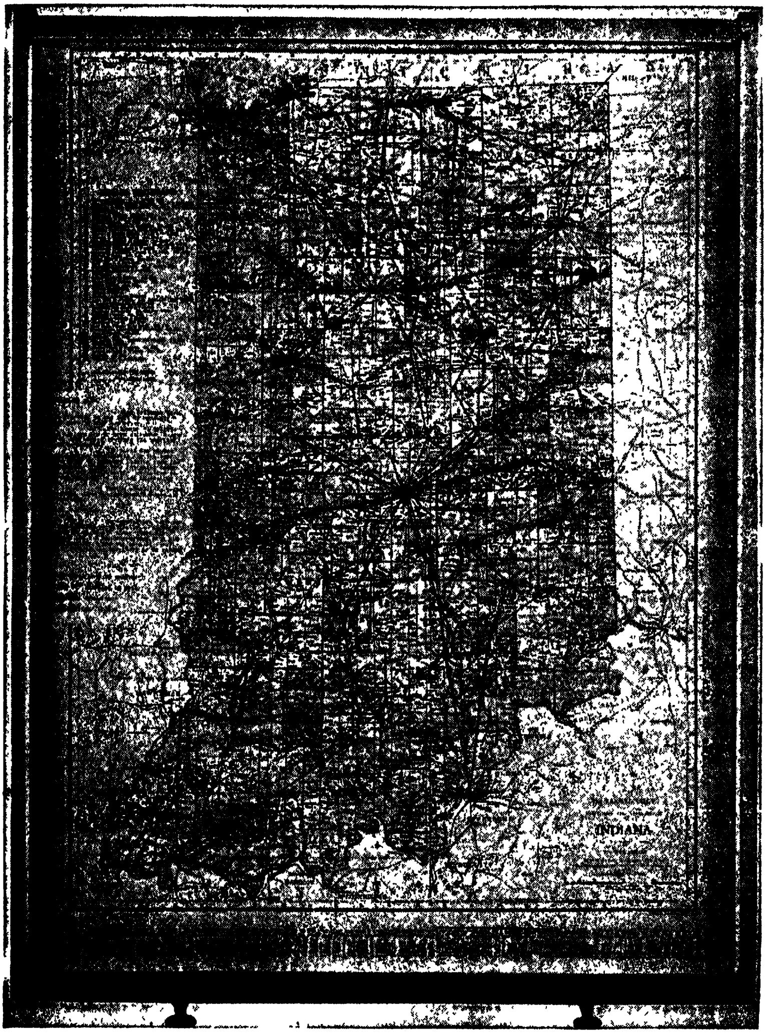
street in the city. In addition to indicating position, the map-tack system may be used to indicate variation in size or quantity.

An example of the vast amount of information which can be



Courtesy of Rand McNally Co.

FIG. 107. MAP TACK GRAPH SHOWING DISTRIBUTION OF AUTOMOBILE SALES



Courtesy of the Rand McNally Co.

FIG. 108. MAP TACK GRAPH SHOWING SALESMEN'S ROUTES

placed on one map-tack graph, resulting in a widespread use of the map-tack system in business, is illustrated in Fig. 107 showing clearly the following information on a single diagram; the distributors and branches of the company; the dealers, by size of dealer; sales and cancellations. For this purpose map tacks or pins may be secured with heads of different sizes, shapes, and colors. A marked degree of flexibility is obtained through the great variety of sizes, shapes, and colors available. Tacks with heads of varying sizes may be used to indicate variations in quantity, while the various shapes and colors may be used to indicate different classes of data, such as types of customers, salesmen, retailers, branches, agents, etc.

The pins on the map may be joined by means of cords to indicate routes such as those followed by salesmen. The flexibility of the map-tack system, due to the interchangeability of the pins on the map, enables it to offer services rendered by no other form of map graph. Changes in quantity and location can be indicated quickly and easily.

The map-tack graph is used primarily as a wall diagram. For this reason, and because of the fact that tacks must easily penetrate a surface and in turn be held tightly, a board or cork mounting must be used. Generally the mounting consists of a layer of cork and two or three layers of soft wood. These mountings may be readily purchased. The size of map-tack systems varies from a small-sized map to room-size graphs, depending on the purpose of the graph, the area to be analyzed, and the density of the data.

CHAPTER XI

ORGANIZATION CHARTS

Equipment for Organization Charts—Planning the Organization Chart—How to Construct an Organization Chart—Classification Charts—Line-of-Flow Chart.

ORGANIZATION charts convert into simple visual form the intricate organization (or routine) of an economic, social, or political body (or process) by indicating the lines of authority, the movement through a process, or a series of necessary steps.

Although organization charts are not graphs in the true sense of the word, since their purpose is not to compare quantities, organization charts have always been included as part of the work of those who construct graphs.

Since these charts quickly and simply summarize complex organizations or movements, they have long been popular, especially in business and in governmental work, and have been widely applied in other fields.

Organization charts may be classified into two major types. The first is commonly known as the organization chart. It illustrates the various units or officials of an organization in visible form, generally with a series of lines to indicate authority and responsibility. The second type is known as the flow (or line-of-flow) chart. This chart is the visual presentation of the series of steps which are followed in a given process.

Organization charts indicate organization or flow by a series of lines passing between a series of points. These points, whether

they represent offices or officials or a step in a process, are generally indicated by means of squares or circles.[1]

Equipment for Organization Charts. Since these charts merely consist of a series of lines and geometric figures, they can readily be constructed with the aid of a ruling pen, a ruler, and perhaps a compass. The lettering may be prepared by hand by any of the methods outlined in Chapter III, or, on the smaller charts, the lettering may be inserted on the typewriter. The simplicity of construction and lack of need of complex equipment have added greatly to the popularity of organization charts.

Planning the Organization Chart. The organization to be presented on the organization chart is generally complex, or there would be no need for such a chart. Therefore, careful planning is necessary before the chart is drawn. Lack of a systematic plan would result in a confused chart.

Care should be taken that the lines in the chart do not unnecessarily cross and recross one another. Repeated crossing and recrossing of lines will make the movement difficult to follow and still more difficult to understand. If, however, it is impossible to avoid the crossing of lines on an organization or flow chart, the lines may be crossed as shown below to indicate that they are connected or not connected.

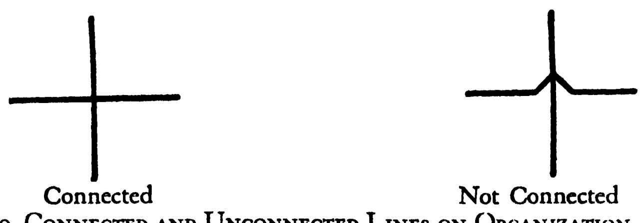


FIG. 109. CONNECTED AND UNCONNECTED LINES ON ORGANIZATION CHARTS

How to Construct an Organization Chart. The chief officer or office of the organization (such as president, board of directors, etc.) is represented by a box or circle drawn at the top center of

[1] The flow chart often appears in pictorial form.

the chart. If there are several chief officers of equal rank they may be represented by a series of equal-sized boxes or circles equally spaced. At a short distance beneath this box (or boxes) the officers or offices of next rank are also represented by boxes. The offices are ranked generally according to authority and responsibility. These boxes are all drawn on a horizontal line and are equally spaced. The center of the distance between the first and last box should be immediately beneath the center of the top box.

The first box may then be joined with those of the second line by means of straight lines. These straight lines may be drawn from the center of the bottom of the first box to the top center of each of the boxes in the second line. If this method is used the lines will radiate from each box to those in the next line. Drawing the connecting lines in this fashion is generally the practice when circles are used rather than squares. The more common practice is to join these boxes by means of rectilinear co-

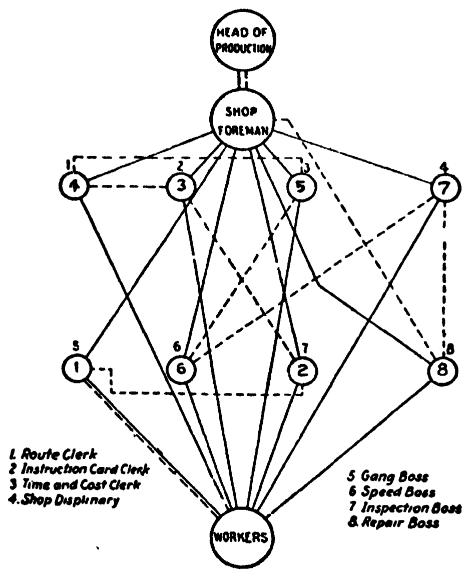
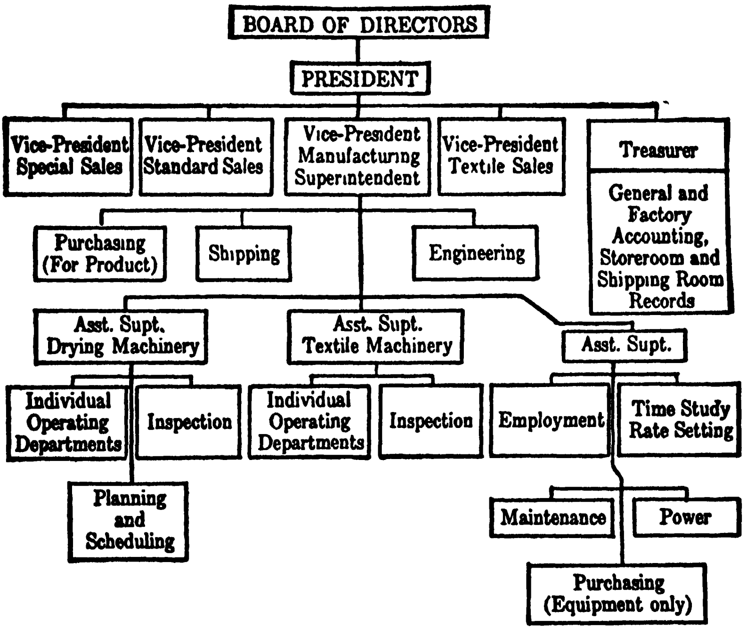


Diagram of modified application of Taylor's functionalized shop managemen

From Wissler, Willis, "Business Administration," p. 88

FIG. 110. ORGANIZATION CHART

ordinates. A short vertical line is dropped from the center of the bottom of the first box. A horizontal line is then drawn touching this line from the center of the first box in the second line to the center of the last box in the row. Short vertical lines are then dropped to the top center of each box in the second row. Further boxes may then be added in rows until all of the offices have been included.



Source: Metropolitan Life Insurance Company, Pamphlet on Business Organization, No. 12, p. 20

FIG. 111. COMPLEX ORGANIZATION CHART—PRODUCTION DEPARTMENT ORGANIZATION

Inter-office connections and responsibilities may be indicated by drawing lines directly connecting the boxes regardless of row.

If desired, the name of the official may be included in the box as well as the name of his office. It is best practice, however, to in-

clude as little as possible in the box so as not to make the diagram difficult to read.

Considerable interest can be added to the organization chart, when it is simple in form and does not include many boxes, by inserting pictures of the various officials holding the several offices.

Classification Charts. If the organization chart presents the systematic arrangement of facts rather than officials or offices of a business, a political or a social organization, it is known as a classification chart. This type of chart presents a logical analysis and arrangement of the facts.

The facts to be presented in a classification chart must permit of classification. It must be possible to arrange the items in divisions and subdivisions.

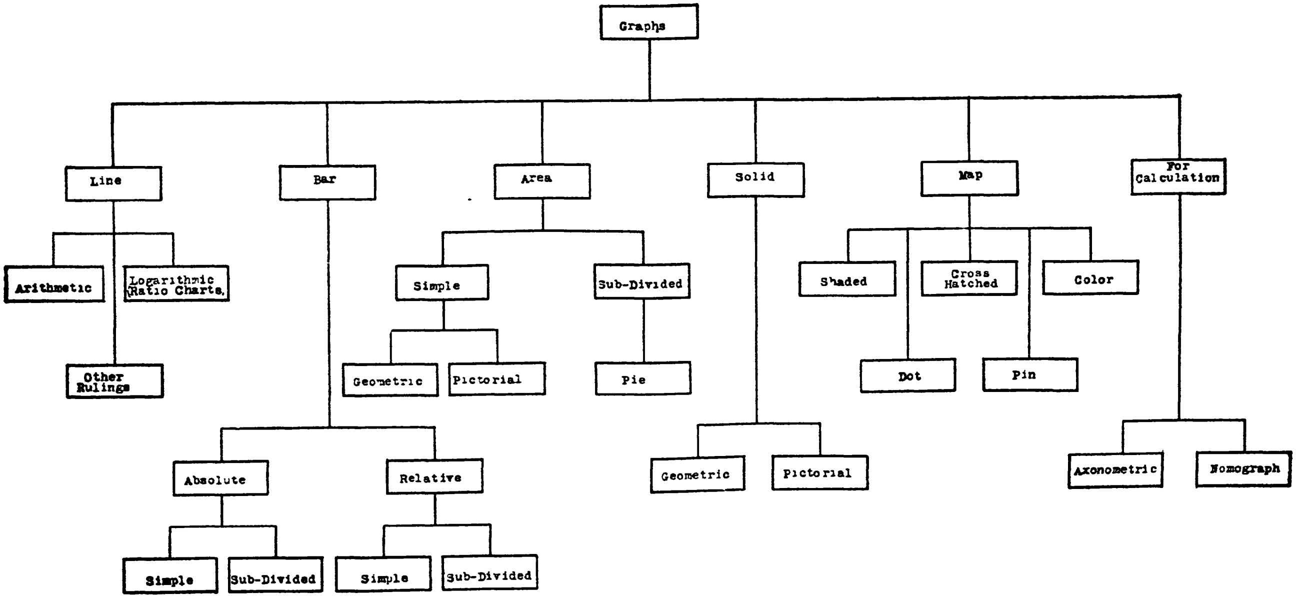
When presented in this form the most complex classification or arrangement of knowledge can quickly be grasped and a greater insight into the subject obtained than by any other method. It is one step further advanced than the outline form as a means of marshalling facts.

The box at the top generally contains the name of the general classification. The major subdivisions are placed in the row immediately beneath the general classification. The subdivisions of these major classifications are then placed in the next row, etc.

Fig. 112 presents a classification chart showing a classification of the types of graphs discussed in this volume.[2]

If there are more boxes than can be fitted into the space allotted to a particular classification, some of the boxes may be dropped to a position below that row. These boxes should be spaced so that they occupy the center of the space between the boxes immediately above it. This arrangement permits of the drawing of the rectangular connecting lines without crossing any boxes.

[2] Not every variation of the major types of graphs in this book were included, however, since this would have made the chart too complex.



Note: Only major types are indicated

FIG. 112. TYPES OF GRAPHS (CLASSIFICATION CHART)

Line-of-Flow Chart. The line-of-flow chart is used to present visually the series of steps followed, for example, by merchandise during a handling process, by a person following some prearranged system, or the presentation of a task from beginning to completion. It is a visual presentation of a fixed series of steps or events.

The chart may be prepared in a number of different ways, ranging from a diagram similar to the organization chart to a pictorial chart.

The simplest of the line-of-flow charts represents each step in a process by means of a box containing a description of the step. These boxes are generally placed in a horizontal row. When there are parallel or alternative steps, other rows paralleling the first may be introduced. These boxes are usually joined by means of straight lines marked with arrows to show the procedure of the steps.

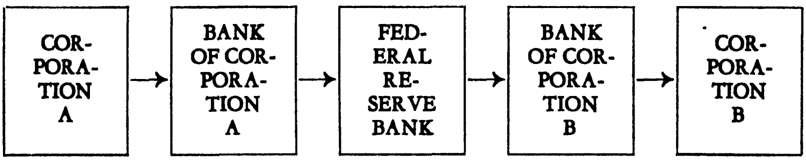
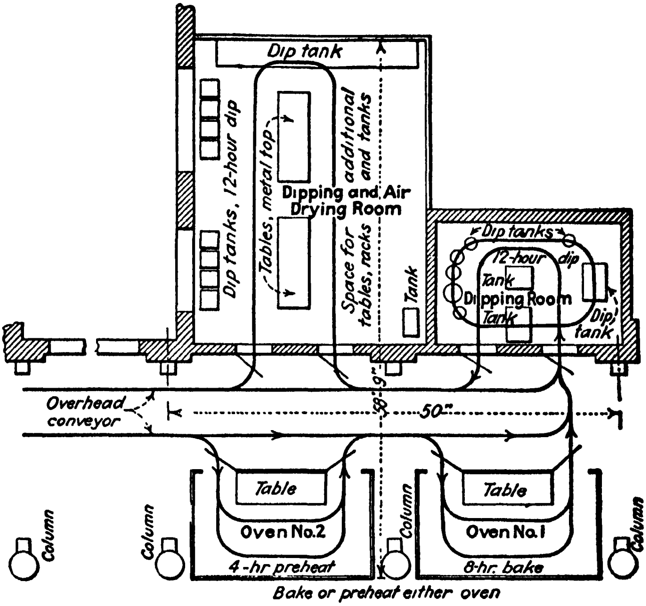


Fig. 113. Clearance of a Check-Line-of-Flow Chart

Another form of line-of-flow chart is based upon a floor plan. This type of chart presents the spatial location as well as the order of the steps. A floor plan is drawn showing the location of the equipment used in the process. The path followed as well as the order of the steps is shown by a series of arrows. Fig. 114 illustrates a diagram of this type.

A more striking form of line-of-flow chart is that which includes pictures of the equipment used in the various steps of a process. These pictures are spaced in the same manner as the line-of-flow chart making use of boxes, but the pictures of the

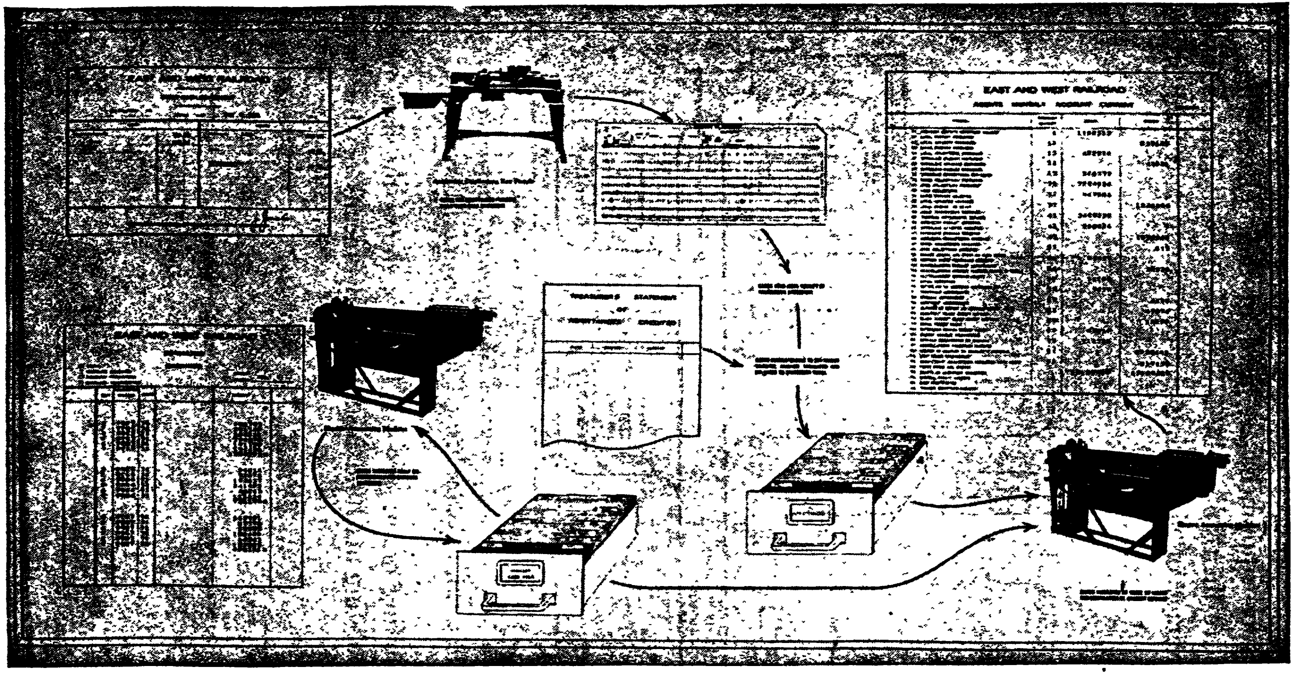
machines or equipment are used in their place. The various pictures are then joined by means of arrows. This method has the advantage of visualizing the equipment and facilitating an



From "Factory and Industrial Management," September 1932

FIG. 114. LINE OF FLOW CHART BASED ON FLOOR PLAN

understanding of the process. Fig. 115 shows the flow of work in a tabulating-machine installation by means of this form of diagram.



Courtesy International Business Machines Corporation

FIG. 115. PICTORIAL LINE OF FLOW CHART

CHAPTER XII

GRAPHS FOR COMPUTATION

The Graph for Computation—How to Construct a Graph for Computing—Nomographs—Principle of the Nomograph—How to Construct a Nomograph—Parallel Nomographs—Non-parallel Nomographs—How to Construct a Non-parallel Nomograph.

THE graph can be used to reduce the time and effort spent in computation. If the computation revolves about a relationship between the values which can be expressed as a formula, repeated calculations for different values requiring substitution in a complex formula can be simplified by the use of a graph of relationship.

The values obtainable by this method, however, are only approximate[1] and, if a very high degree of accuracy is desired, a graph constructed for computing purposes cannot be used successfully.

Frequently it is necessary in financial work to compute the yield of a bond, given its price. Since the interest rate and maturity of the bond are available, its yield can be computed from a formula. This computation is rather difficult. To minimize the calculations necessary, it is possible to construct a series of graphs of relationship for the more common interest rates and maturity periods, showing the relationship between yield and price. A graph of this type for a bond with an interest rate of 5 per cent, payable semiannually, and a maturity of twenty years is shown

[1] The accuracy of the result obtained is dependent upon the size of the graph and the number of background rulings.

in Fig. 116. Similar curves may be drawn on the same graph for other common interest rates. On the graph in Fig. 116, lines for interest rates of 4 per cent and 6 per cent have also been included.

If it is desired to obtain the yield of a bond with a 5 per cent interest rate, payable semiannually, maturing in twenty years, and

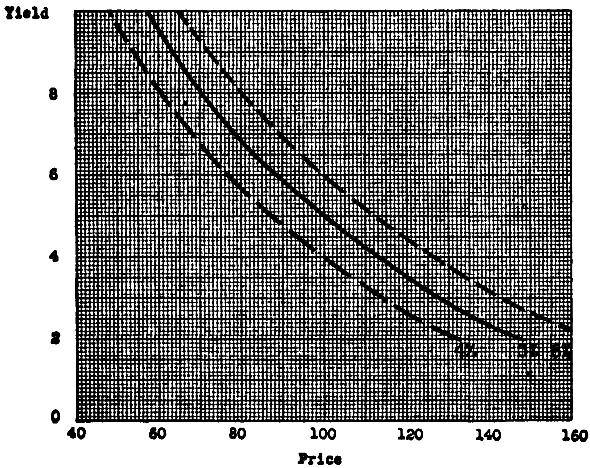


FIG. 116. GRAPH FOR COMPUTING BOND YIELDS FOR A 20-YEAR MATURITY

selling at a price of 94, the sales price is located on the X axis and a vertical line is drawn to meet the curve. A horizontal line is drawn from this point on the curve, meeting the Y axis. The value of the point where the horizontal line intersects the Y axis will be the yield, in this instance approximately 5.50 per cent.

How to Construct a Graph for Computing.

1. Determine the formula for the relationship. The formula for the electrical resistance of a copper wire of 100-per-cent conductivity at 20° C. is:

$$R = 10.371 \frac{\text{length in feet}}{\text{Section in circular mils}}$$

2. The values corresponding to various thicknesses of wire of

1 foot in length may now be computed. The table below shows the resistance of various thicknesses of copper wire.

RESISTANCE PER FOOT OF COPPER WIRE OF 100-PER-CENT CONDUCTIVITY AT 20° C.

Section* in Circular Mils	Resistance in Ohms
10	1 037
20	.519
30	.346
40	.259
50	.207
60	.173
70	.148
80	.130
90	.115
100	.104

* A circular mil is the area of a circle one mil in diameter (one thousandth of an inch).

3. An arithmetic grid is prepared. Numerous background rulings and a finely divided scale of values should be inserted, since the grid is to be used to determine values rather than merely present data. Printed graph sheets may be used for this purpose.
4. The values in the above table are plotted on the grid and a *smooth* curve drawn joining the various points. The finished graph appears in Fig. 117.

Nomographs. A limitation exists in the use of the type of graph presented above in that only two variables can be handled at one time. Thus, in the bond yield illustration (see Fig. 116) only two variables (in this case bond prices and bond yields) were dealt with in one graph. Since the interest rates and the number of years to maturity will vary from bond to bond, a large number of graphs would be necessary to permit extensive calculation. It would be necessary to have one graph for every maturity which

might occur, and numerous lines on each graph for every possible interest rate.

To determine the resistance of a copper wire, the procedure for which is outlined above, it is necessary to have a large number of graphs or a large number of curves on one graph, since different temperatures result in different resistances. Only the thickness of

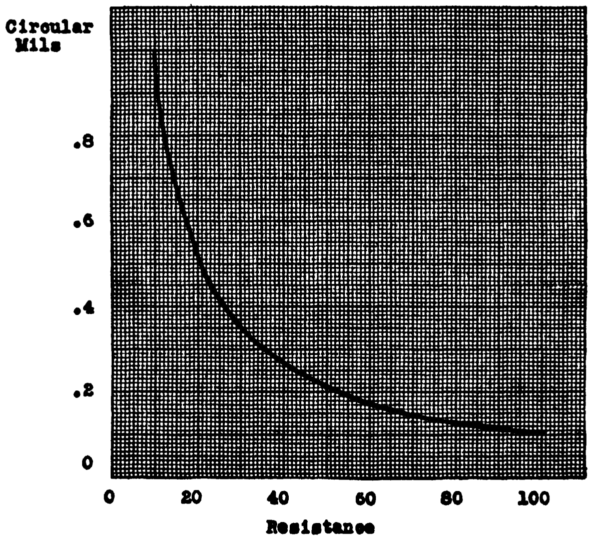


FIG. 117. RESISTANCE PER FOOT OF COPPER WIRE OF 100 PER CENT CONDUCTIVITY AT 20° CENTIGRADE

the wire and its relation to its electrical resistance is covered by this graph.

By means of the *nomograph*[2] more than one variable can be handled. The usual nomograph deals with two variables at one time. The nomograph consists of a series of scales placed side by side. If a line is drawn through two of the scales, a computed result can be determined from the third. A simple nomograph which may be used to obtain the loss of power in an electric circuit is shown in Fig. 118.

[2] The nomograph is also known as a nomogram or alignment chart.

Given a circuit with a resistance of 1 ohm and a current of 10 amperes passing through it, the loss of power can be obtained by locating 1 on the resistance scale (point x), 10 on the height scale (point y), and drawing a straight line through the two points. The value on the volume scale at the point (point z) at which the drawn line intersects the power loss scale is the loss of power (in this instance 100 watts).

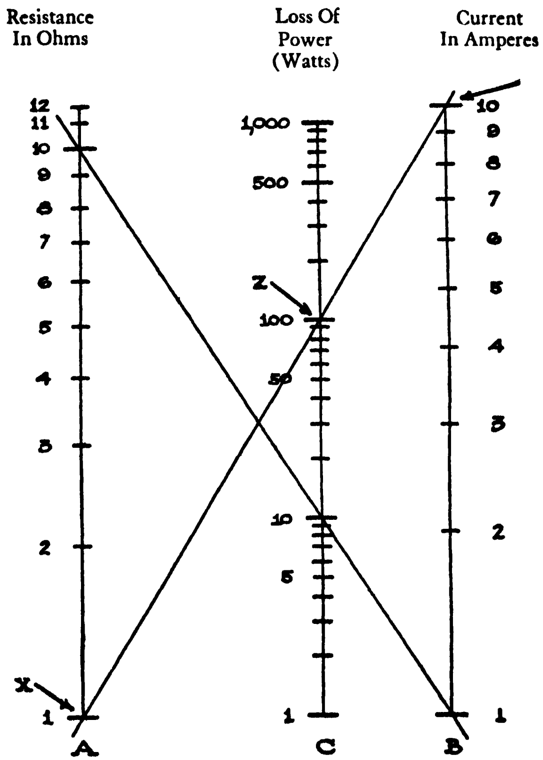


Fig. 118. Nomograph for Computing the Loss of Power in an Electric Circuit

Principle of the Nomograph. If two identical arithmetic scales of the same length are placed parallel to one another (scales A and B in Fig. 119, with values ranging from zero to 10) and an arithmetic scale of the same length with double the values (scale C

in Fig. 119 with values ranging from 0 to 20) is placed halfway between, the result is a nomograph for addition. If a line is drawn connecting any two points on scales A and B, the sum of the two values will be indicated at the point where this line intersects scale C. Thus line X drawn from 4 on the A scale to 4 on the B scale intersects the C scale at 8 ($4+4=8$). Line Y passes from 5 on the A scale to 7 on the B scale and intersects the C scale at 12 ($5+7=12$).

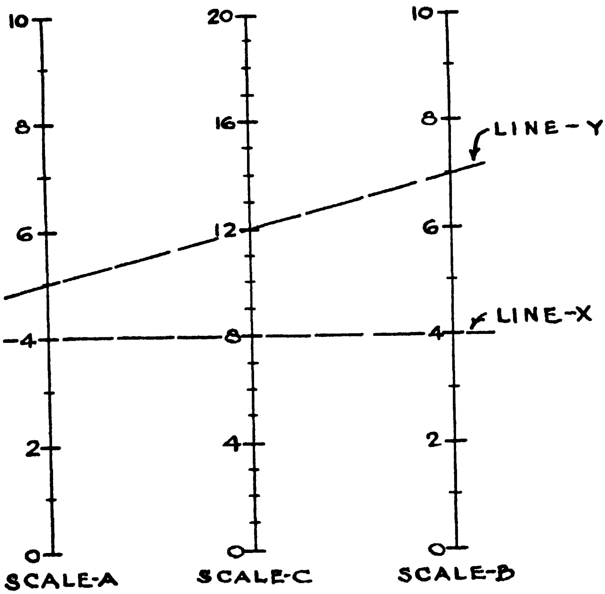


FIG. 119. ADDITIVE NOMOGRAPH

If Scales A and C are used as the reference scales rather than A and B, the resulting computation is a subtraction. If 8 is to be subtracted from 18, a line (line W in Fig. 120) is drawn from 8 on the A scale through 18 on the C scale and extended to the B scale. The remainder (in this case 10) will be indicated at the point of intersection on the B scale.

Since the addition and subtraction of two figures is a simple task, no great saving in the process can be accomplished through

the use of a nomograph. However, nomographs can be used for multiplication and division through the use of logarithms. When two numbers are to be multiplied their logarithms are added. If the nomograph is constructed with logarithmic scales and used as an addition nomograph, the result is a product obtained by adding logarithms. In Fig. 121, the multiplication of 4 and 5

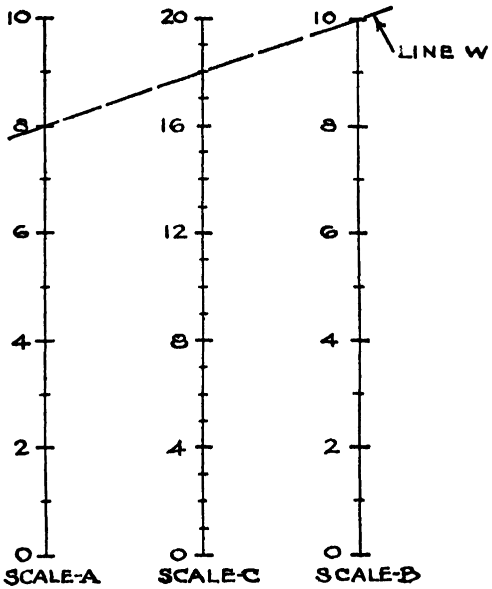


FIG. 120. NOMOGRAPH FOR SUBTRACTION

is accomplished by locating 4 on logarithmic scale A, 5 on logarithmic scale B, and drawing a line to connect the points. This line (line X, Fig. 121) intersects scale C at the product of the two numbers.

In a like manner division may be accomplished, since a dividend is obtained by subtracting the logarithms of two numbers. If 8 is to be divided by 2, the nomograph with logarithmic scales is used for subtraction. Thus 8 is located on logarithmic scale C, and 2 on scale A. The line (line Y, Fig. 121) connecting these two

points is extended to intersect scale B. The point of intersection indicates the dividend.

The simple additive nomograph may be explained as $C = A + B$. To obtain this result it is important that the C-scale values be double those of the A and B scale within a scale of the

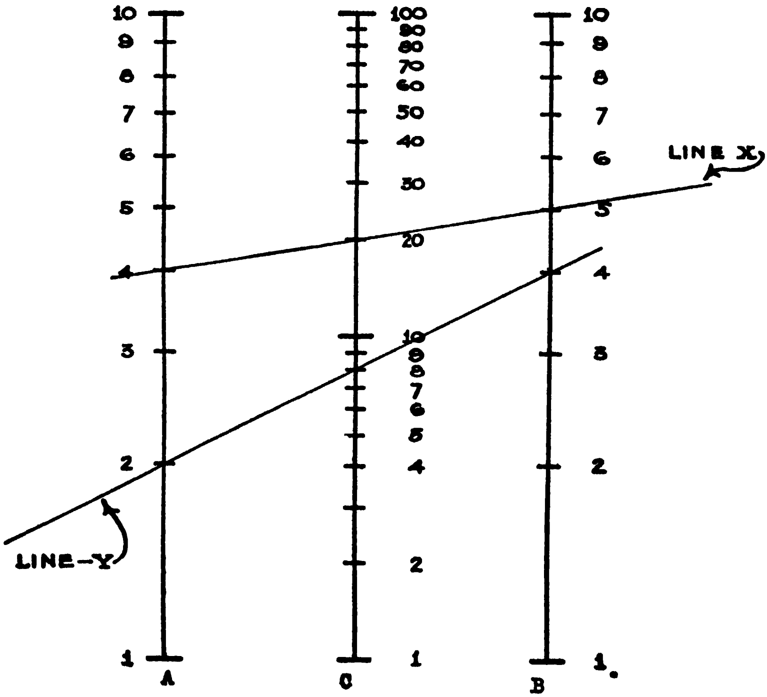


FIG. 121. MULTIPLICATION NOMOGRAPH

same length. If the scale units are the same, the result will be $C = \frac{A + B}{2}$ (see nomograph 1 in Fig. 122). If either of the other scales should be doubled, within a scale of the same length, for example the B scale, the result would be $C = A + \frac{B}{2}$. Increasing or decreasing the scales A or B causes the quantity on the scale to be added as only a fraction of that value. Many possible varia-

tions can thus be obtained by increasing or deceasing the number of units on the various axes, while retaining scales of the same lengths. A few of the results possible are indicated below:

If

SA is the number of units on the scale on the A axis.

SB is the number of units on the scale on the B axis.

SC is the number of units on the scale on the C axis.

When

Scale Relation Is	Resulting Computation When Answer Is Read from	
	C Scale	B Scale
1. $S_A = S_B = 2S_C$	$C = A + B$	$B = C - A$ (See Nomograph 1, Fig. 122)
2. $S_A = S_B = S_C$	$C = \frac{A+B}{2}$	$B = 2C - A$ (See Nomograph 2)
3. $S_A = S_B = 4S_C$	$C = 2(A + B)$	$B = \frac{1}{2}C - A$ (See Nomograph 3)
4. $S_A = 2S_B = 2S_C$	$C = A + \frac{1}{2}B$	$B = 2(C - A)$ (See Nomograph 4)
5. $S_A = \frac{1}{2}S_B = 2S_C$	$C = A + 2B$	$B = \frac{1}{2}(C - A)$ (See Nomograph 5)
6. $\frac{1}{2}S_A = S_B = 2S_C$	$C = 2A + B$	$B = C - 2A$ (See Nomograph 6)

It can be seen from this table that doubling the scale is equivalent to dividing by 2 the number added or subtracted. In a similar manner, tripling the scale would be equivalent to dividing by 3, the value added or subtracted, etc. If the number of units contained within the given distance allotted for the scale is doubled, the values added or subtracted by use of the scale will be halved.

To multiply a given value by 3, only one-third of the scale values would be used over the same length. When all the scales are equal in length and scales A and B contain the same number of units, while C contains twice as much, the value read off from the C scale will be A + B. To obtain the sum of 3A and 4B, as compared with the ordinary additive nomograph (A + B), the A scale would show one-third as many scale divisions, while the B scale would have one-fourth as many. For the sum of ¼A and

2B, the A scale would have 4 times as many units as on the additive nomograph, while the B scale would have half as many.

Raising a number to a given power, or taking a given root of a number, is accomplished by multiplying the logarithm of the number, or dividing the logarithm of the number by the power or

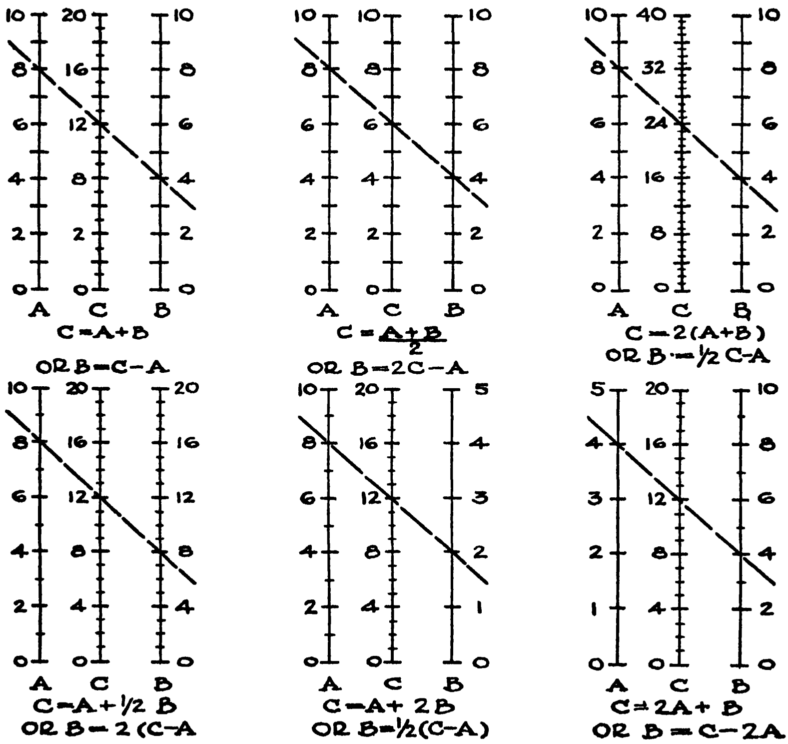


FIG. 122. ADDITIVE NOMOGRAPHS

root. Thus, the square of a number may be obtained by multiplying the logarithm of the number by 2, and the square root by dividing the logarithm by 2.

If logarithmic scales are used in preparing the nomographs, not only multiplications and divisions can be performed as explained previously, but the multiplied or divided figures can be raised to a power by varying the scales. For instance, $C = AB^2$, in

logarithmic form, is log C = log A + 2 log B. Thus, by making use of logarithmic scales and varying the scales as in 5 in the table above, this logarithmic addition, and consequently the multiplication, will result. Some possible products resulting from variations of the scale are listed below:

If

S'_a is the number of units on the logarithmic scale on the A axis.

S'_b is the number of units on the logarithmic scale on the B axis.

S'_c is the number of units on the logarithmic scale on the C axis.

When

Scale Relation Is	Resulting Computation When Answer Is Read from	
	C Scale	B Scale
1. $S'_A = S'_B = 2S_c'$	$C = AB$	$B = \frac{C}{A}$ (See Nomograph 1, Fig. 123)
2. $S'_A = S_B' = S_c'$	$C = \sqrt{AB}$	$B = \frac{C^2}{A}$ (See Nomograph 2, Fig. 123)
3. $S'_A = S_B' = 4S'_c$	$C = (AB)^2$	$B = \frac{\sqrt{C}}{A}$ (See Nomograph 3, Fig. 123)
4. $S'_A = 2S_B' = 2S'_c$	$C = A\sqrt{B}$	$B = \frac{(C)^2}{(A)}$ (See Nomograph 4, Fig. 123)
5. $S'_A = \frac{1}{2}S_B' = 2S'_c$	$C = AB^2$	$B = \sqrt{\frac{C}{A}}$ (See Nomograph 5, Fig. 123)
6. $\frac{1}{2}S'_A = S_B' = 2S'_c$	$C = A^2B$	$B = \frac{C}{A^2}$ (See Nomograph 6, Fig. 123)

As previously shown, increasing the number of units on the scale on the addition nomograph has the effect of dividing the value added or subtracted by the proportion of the increase. Decreasing the number of units on this type of diagram, within the same scale length, has the effect of multiplying the value subtracted in proportion to the decrease. Since multiplication or division is accomplished by adding or subtracting logarithms by use of the nomograph, if it is desired to raise any value to a

given power in the product, or to use its root, the result can be obtained by adjusting the scale.

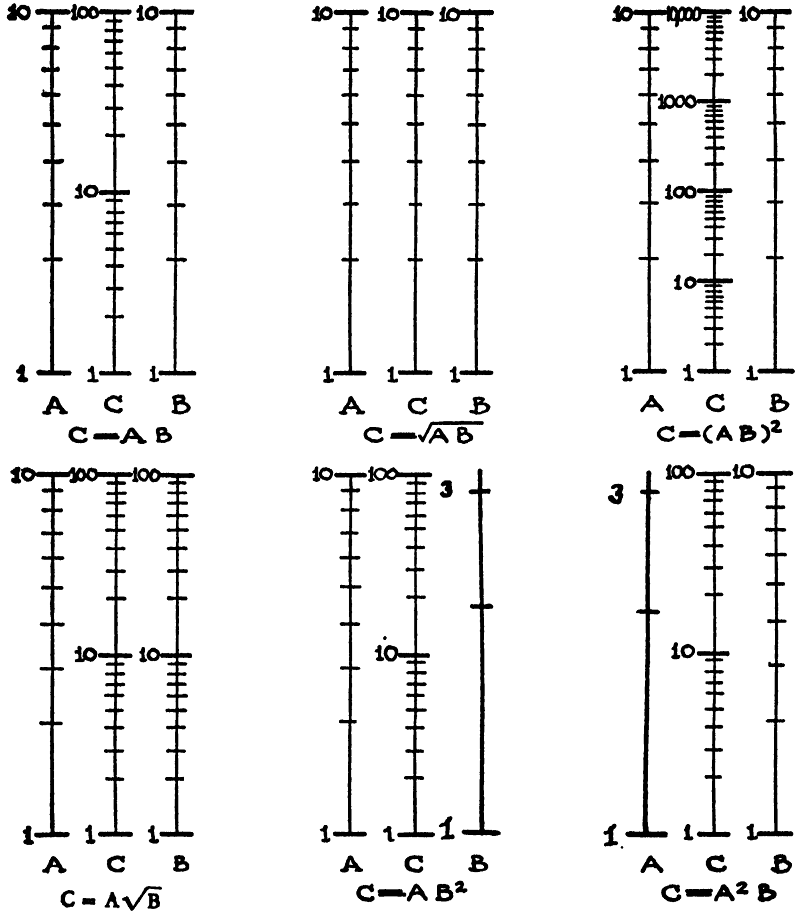


FIG. 123. NOMOGRAPH FOR MULTIPLICATION AND DIVISION

Reducing by half the number of scale units used on the given length of scale has the effect of multiplying the value added, and when a logarithmic scale is used the value is a logarithm. This operation would have the effect of multiplying the logarithm by

two. In this way the original number is squared while adding it to the other logarithm, thus resulting in the desired product.

In the additive nomograph, the A and B scales contained the same number of units, while the C scale had twice as many within the same length. A multiplication or division nomograph has the same relation between the scales, with the exception that the scales are in logarithmic form. Scales A and B would have the same number of logarithmic tiers or cycles, while scale C would have twice as many within the same length. To obtain the product of A and B^2, the number of cycles on the scale A would be doubled, while those on the B scale are halved, as compared with the simple multiplication nomograph. Thus the value of the logarithm of the A value added would be halved, while the logarithm of the B value would be doubled.

Similar variations in results may be obtained by constructing the number of scale divisions and varying the distance of the central scale from the others. If the number of units on the scale is uniform for *all* scales (A, B, and C) the sum obtained will vary in accordance with the following formula

$$C = \frac{d_2}{D} A + \frac{d_1}{D} B$$

where

d_1 is the distance from scale C to scale A
d_2 is the distance from scale C to scale B
D is the total distance between the two outer scales.

Nomograph 2 on page 171 shows three uniform scales of the same length, with the C scale halfway between the A and B scales. If the total distance between the two scales is considered as 2 units, the C scale is 1 unit from each of the other axes, and d_1 and d_2 are each equal to one unit.

The formula then reads:

$$C = \tfrac{1}{2}A + \tfrac{1}{2}B = \frac{A + B}{2} \text{ or for subtraction } B = 2C - A.$$

In Fig. 125 the C scale is 1 inch from the A scale and 2 inches from the B scale. The value of d_1 is therefore 1, while d_2 is 2.

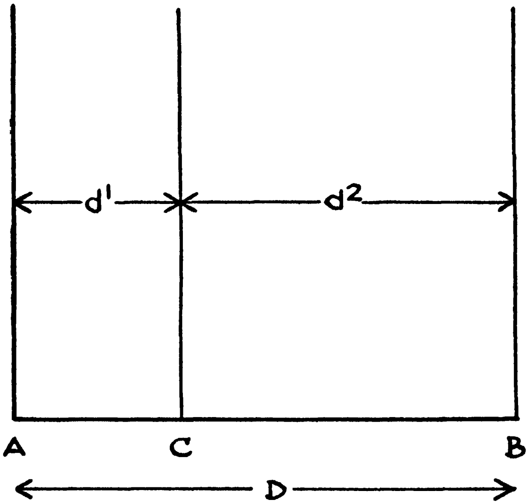


FIG. 124. THE AXES OF THE NOMOGRAPH

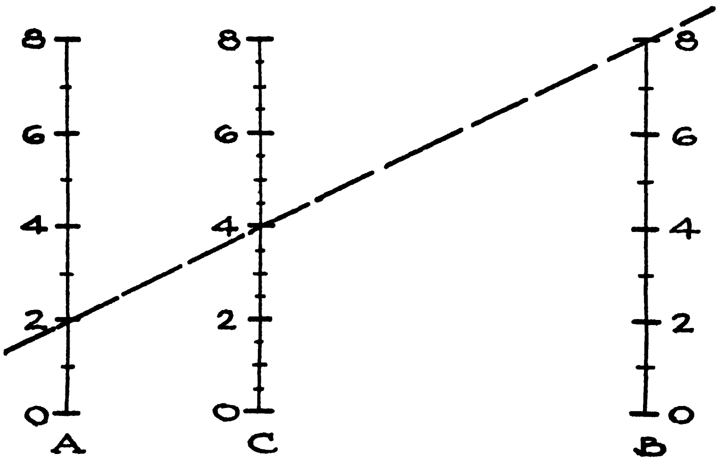


FIG. 125. ADDITIVE NOMOGRAPH

Substituting in the formula:

$$C = \tfrac{2}{3}A + \tfrac{1}{3}B = \frac{2A + 1B}{3}$$

or, if used for subtraction,

$$B = 3C - 2A$$

The values on the A and B scales may thus be included in the sum any number of times (or be multiplied by any fraction) by shifting the C axis to the appropriate position between the other two scales. If the scales are logarithmic, the logarithm may be multiplied by any fraction (divided by any number) while adding it to the other logarithm, thus obtaining any root of the original number while securing the product.

The product $C = \sqrt[3]{AB^2}$, which may be written $C = A^{\frac{1}{3}}B^{\frac{2}{3}}$, can be obtained by using uniform logarithmic axes with the C scale one-third of the distance between the outer scales away from the B axis, and two-thirds of the distance away from the A axis.

The same result ($\sqrt[3]{AB^2}$,) may be obtained by decreasing the number of units on the B scale of the ordinary multiplication nomograph by one-half, while retaining the same scale length (to obtain AB^2, see Nomograph 5, page 173) and reducing the C scale to one-third of its usual size. In terms of logarithms this results in

$$3 \log C = \log A + 2 \log B$$

or

$$\log C = \frac{\log A + 2 \log B}{3}$$

or

$$C = \sqrt[3]{AB^2}$$

A constant appearing in the equation can be added to the sum (or minuend) or the product can be multiplied (or divided) by the constant through adding or multiplying one of the scales by the amount of the constant. If $A + B + 2$ is to be contained on the C axis 2 is added to the scale values on the C axis. If 2AB is to be obtained, the values on the C scale may be multiplied by this constant.

There are also nomographs, in which the scales are not parallel, used to express more complex relations. However, such nomographs are beyond the scope of this volume.

How to Construct a Nomograph. Thus, the construction of the nomograph is dependent on two factors: (1) the proportionate distance of the C axis from the A and B axes, and (2) the number of units on the given length on each scale, or the scale modulus.[8]

1. Determine the formula for the relationship between the two variables. For example, the formula for the loss of power in an electric circuit is

$$L = 2I^2r$$

where L = loss of power
I = current in amperes
r = resistance of circuit in ohms

This formula is reduced to its logarithmic form

$$\text{Log } L = \log 2 + 2 \log I + \log r$$

The constant (2) temporarily may be dropped from consideration and included later.

2. Determine the range of values for the independent variable for which computation will be desired. In this instance the desired limits are from 1 to 12 ohms for the resistance of the circuit and from 1 to 10 amperes for the current passing through it.
3. Draw two parallel axes. The axes should be long enough to make possible scale graduations fine enough to give the desired degree of accuracy. A good relation between the length of these lines and the distance between them is generally approximately one for the height to two-thirds for the space between them. For this nomograph, two axes 5 inches in height and 3 inches apart were drawn.

[8] The scale modulus may be defined as the number of inches per unit of scale. On the logarithmic scale, the scale unit is taken as the distance occupied by one phase or cycle.

4. Prepare the scales on these axes. Since this nomograph is of the multiplication type, logarithmic scales are arranged. These scales will start at the lowest desired figure and range to the highest. The size of the cycle to be used for logarithmic scales is determined by dividing the length of the scale by the difference between the logarithms of the highest and lowest figures. The resistance scale (A scale) was 5 inches in length. The lowest figure desired on this scale is 1 ohm, while the highest is 12 ohms.

   The logarithm of 12 is 1.07918, while that of 1 is zero. The difference between the two logarithms is therefore 1.07918. The length of the cycle to be used is obtained by dividing this figure into the length of the scale, 5 inches divided by 1.07918, or 4.633 inches. The size of the cycle for the B axis is obtained in the same fashion (5 inches divided by 1.0000, or 5 inches). Scales with cycles of this size may now be marked off on the axes, and starting with the lowest value required (in this case 1 for both axes). This can be accomplished by use of a flexible logarithmic scale (see page 77), printed logarithmic sheets, or a slide rule, which consists of a number of such scales of various sizes, or with the aid of a set of logarithm tables.

5. The C axis may now be located. Its position may be determined from the formula

$$\frac{d_1}{(D - d_1)} = \frac{m_a}{m_b}$$

where

d_1 is the distance of the C scale from the A scale.
D is the distance between the A and D scales.
m_a is the scale modulus on the A scale—distance on scale occupied by 1 unit (for logarithmic scales the height of a cycle since the difference between the logarithms of the powers of ten with which each cycle begins is 1).
m_b is the scale modulus for the B scale.

The computation for this problem is as follows: m_a the scale modulus for the A scale was computed above and is 4.633. The value on the B scale is squared in this computation and therefore its logarithm must be doubled. To accomplish this, the scale must be halved or the scale modulus divided by two (or $\frac{5}{2}$).

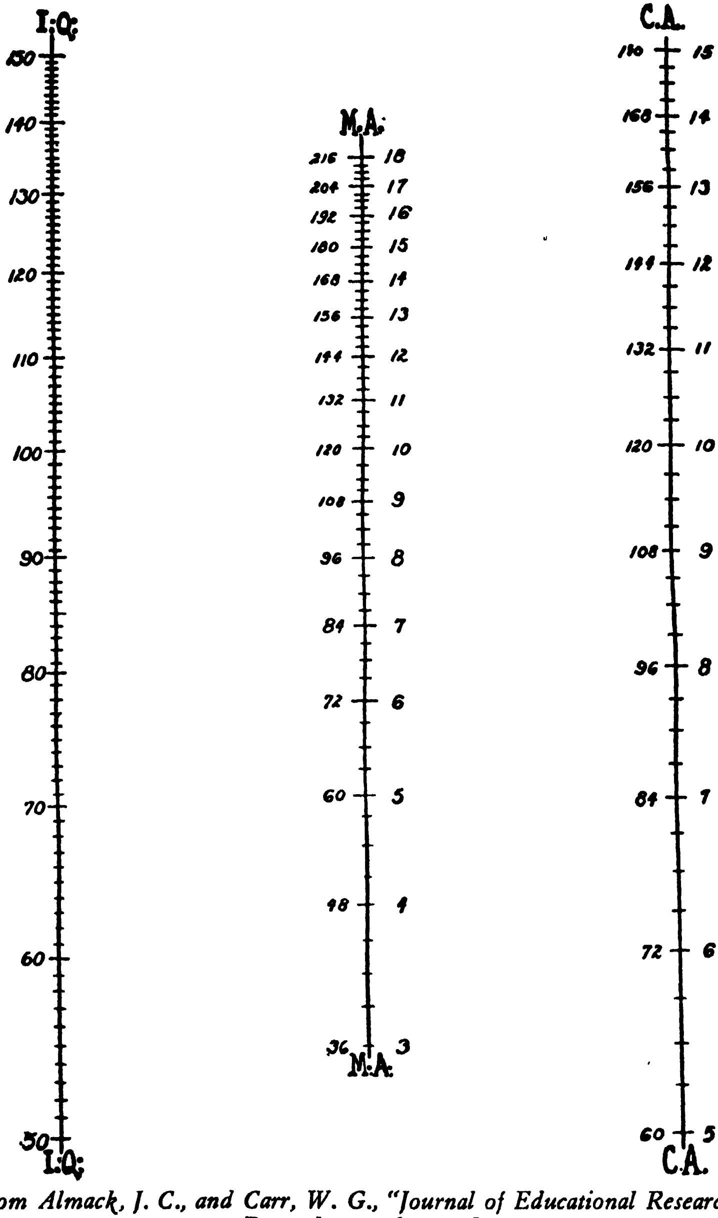
$$\frac{d_1}{3 - d_1} = \frac{4.633}{\left(\frac{5}{2}\right)} \text{ or } d_1 = 1.95$$

The C scale is erected at a point 1.13 inches from the A scale. The same computation can be performed for additive nomographs. The modulus in this instance is obtained by dividing the total range of the scale into its length.

6. The scale is now determined for the C axis. Since this is a multiplication nomograph, the scale will be logarithmic. Its lowest value can be determined (disregarding the constant 2) by using the two lowest values on the A and B scales (1 in both cases) and substituting in the formula I^2r, obtaining 1. The lower limit of the C scale is placed at the position indicated by a line drawn between the lowest points on the A and B scales (when A = 1, and B = 1, C equals 1). The constant 2 must be taken into consideration, however, and therefore this figure is multiplied by that constant (1x2) and the result (2) used as the lower limit.

   Since the form of the cycle is generally known if the size of the cycle can be determined, it is a comparatively easy task to complete the nomograph. The size of the cycle (scale modulus) for the C axis can be determined from:

$$m_c = \frac{m_a m_b}{(m_a + m_b)} \quad \text{or} \quad m_c = \frac{\left(4.633\right)\left(\frac{5}{2}\right)}{\left(4.633 + \frac{5}{2}\right)} = 1.62$$



From Almack, J. C., and Carr, W. G., "Journal of Educational Research," December 1926, p. 348

FIG. 126. NOMOGRAPH FOR THE COMPUTATION OF THE INTELLIGENCE QUOTIENT

Given this height, it is now a simple task to erect a logarithmic scale with a cycle of 1.62 inches. Since the result must include the constant 2, all of the scale values must be multiplied by its value. The answer obtained before doing this is I^2r, while $2I^2r$ is desired. The completed nomograph is shown in Fig. 118.

The nomograph in Fig. 126 may be used for the computation of intelligence quotients. Although the formula is simple in this instance, the nomograph saves considerable time, due to the necessity for frequent repetition of the computations.

Figure 127 illustrates a nomograph useful in engineering and in industry. This nomograph facilitates the calculation of the contents of partially filled horizontal tanks with bulged ends.

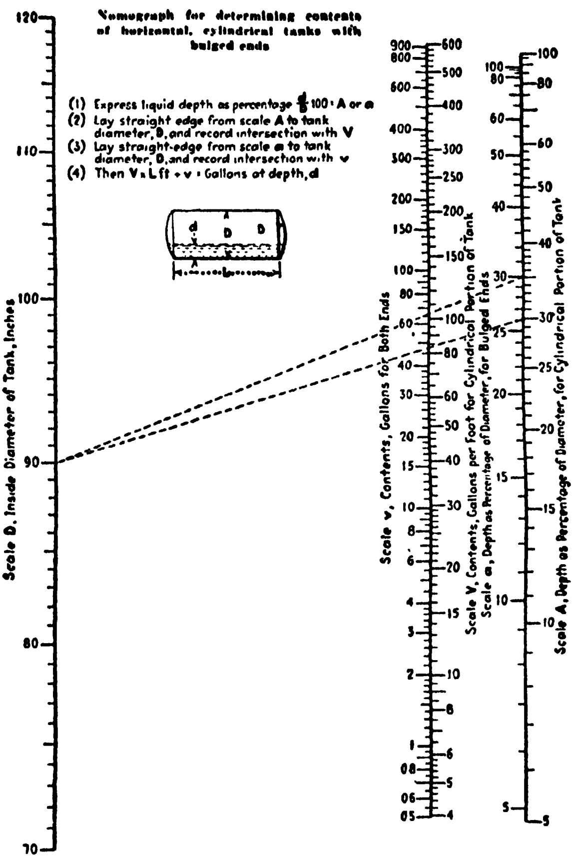
Non-Parallel Nomographs. For the solution of many of the more complex formulas, the axes of the nomograph are not parallel and may even be curved.

The simplest method for constructing a nomograph to solve complex formulas is to set up two conveniently spaced axes and determine by the location of the different values the form and position, as well as the location, of the axes and the scale values. The nomograph for bond yields for a twenty-year bond, with interest payable semi-annually, was constructed in this fashion (see Fig. 129).

How to Construct a Non-Parallel Nomograph. The nomograph for bond yields for a bond with twenty years to maturity, interest payable semi-annually, will be used to illustrate the procedure. The two variables are the market price and the interest rate.

1. Erect two parallel axes at a convenient distance apart. For the bond yield nomograph, two parallel axes 5 inches apart were drawn.
2. Arrange an appropriate scale on the two axes. The range

of values which was used on the A axis for the market price was from 80 to 100. For the B axis, used for the



Reproduced from Davis, D. S., Nomograph for Contents of Horizontal Tanks with Bulged Ends, "Chemical and Metallurgical Engineering," November, 1934

FIG. 127. A PARALLEL NOMOGRAPH

interest rate, the range was from 3 per cent to 8 per cent. Since a low interest rate is commonly accompanied by a high market price, and vice versa, the market price (A) scale was inverted. With this inversion 80 was at the top

of the A scale and 110 at the bottom, while 8 per cent was at the top of the B scale and 3 per cent at the bottom.

3. The C scale (bond yield scale) is then located and its form determined. From the formula, or from a set of tables, two sets of interest rates and market prices with the same yield are determined. For instance, an interest

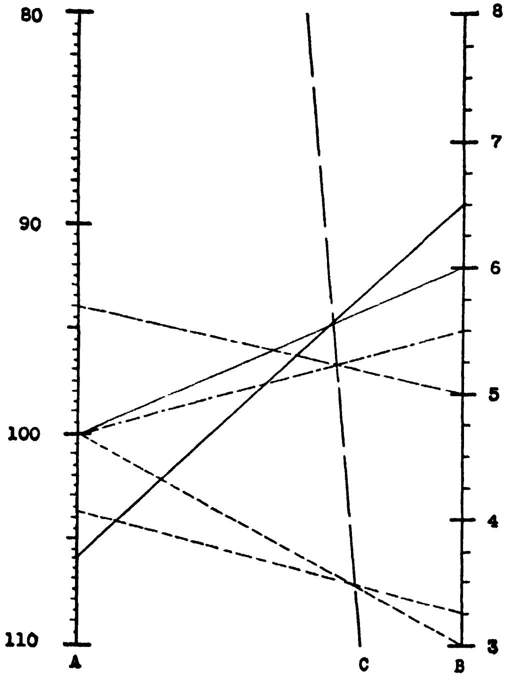


Fig. 128. Construction of a Non-parallel Nomograph

rate of 5½ per cent accompanied by a market price of 100 results in the yield of 5½ per cent, while an interest rate of 5 per cent with a market price of 94 will result in the same yield (5½ per cent).

A line was drawn from 100 on the A scale (market-price scale) to 5½ per cent on the interest scale (B scale). Another line was then drawn from 94 on the A scale to 5

per cent on the B scale. The C axis is located at the point of intersection. That point on the scale is 5½ per cent. Lines were then drawn from 100 to 6 per cent and from 106 to 6½ per cent. Both of these sets of values also have

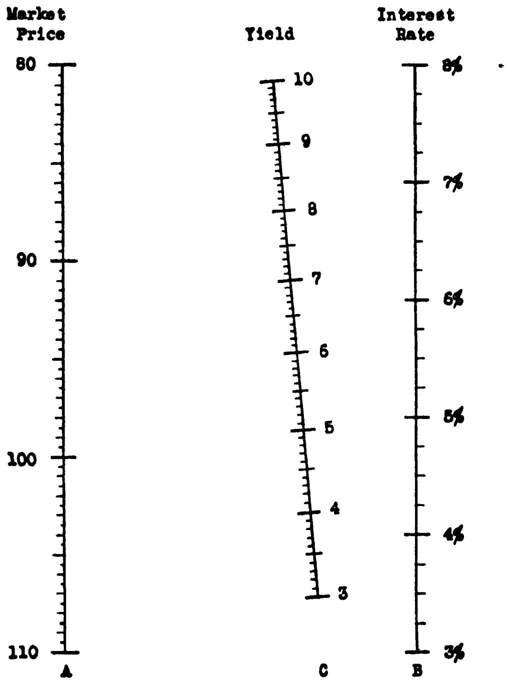
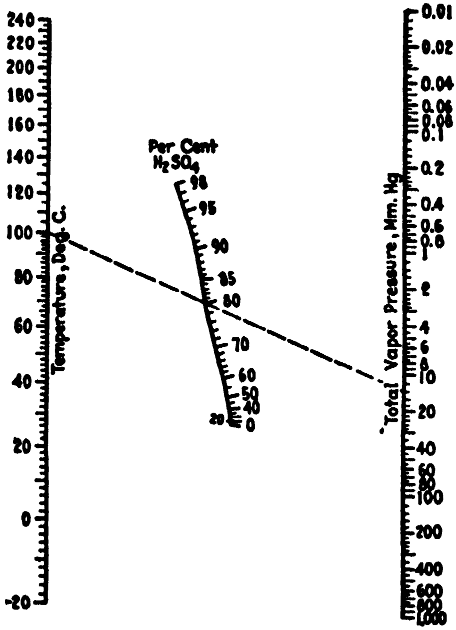


FIG. 129. A NOMOGRAPH FOR THE COMPUTATION OF BOND YIELDS

the same yield (6 per cent). The point of intersection of these two lines marked another point on the C axis (6 per cent). This was repeated for 100 and 3, and 103 and 3¼ per cent both with a yield of 3 per cent

A straight line was drawn through these three points of intersection. This was the C axis. The scale divisions on the C axis were then determined by drawing lines from

various market prices to various interest rates and indicating the resulting values on the C scale.



From "Chemical and Metallurgical Engineering," February 1934

FIG. 130. NOMOGRAPH FOR COMPUTING THE TOTAL VAPOR PRESSURE OF AQUEOUS SULPHURIC ACID SOLUTIONS

A non-parallel nomograph with a curved axis is illustrated in Fig. 130, which is a nomograph for computing the total vapor pressure of aqueous sulphuric acid solutions.

CHAPTER XIII

CONTROL CHARTS

Control Charts—Principle of the Gantt Chart—Construction of the Gantt Chart—Grid—Symbols—Legend—Machine Record Chart—Other Gantt Charts—Man Record Chart—Load Chart—Shipping Chart—Layout Chart—Advantages of the Gantt Chart—The Break-Even Point Chart—The Zee Chart.

Control Charts. There are a number of special types of graphs which are used for the purpose of control. Just as the engineer at the power house or the pilot of the airplane watches a multitude of instruments to determine if all is functioning well, the executive may watch the functioning of every part of his intricate business enterprise through the use of control charts and graphs supplemented by statistical and accounting reports.

Control charts, such as the Gantt Chart for production control, the Break-Even Point Chart for management control, and the Zee chart for sales, etc., control, are discussed in this chapter.

In addition to these and other control charts, the types of graphs which already have been discussed in great detail may serve a similar purpose.

The Gantt chart is a specialized type of control graph which is used as a performance record.[1] In form it is a special type of horizontal bar chart. The Gantt chart is commonly used to portray progress by comparing actual to expected performance for a given period of time, and also to present the cumulative performance over a period of time.

[1] The Gantt chart is named after its creator, Henry Laurence Gantt, a famous management engineer.

Principle of the Gantt Chart. In Fig. 131 a horizontal space is allotted to the time interval, in this case one month, January. The total space allotted to that interval is used to represent 100 per cent of the scheduled performance. The bar that is drawn represents the actual performance during the month. Thus if, for the first month, 50 per cent of the quota has been accomplished, a bar one-half the length of the space is drawn in that space (see Fig. 131).

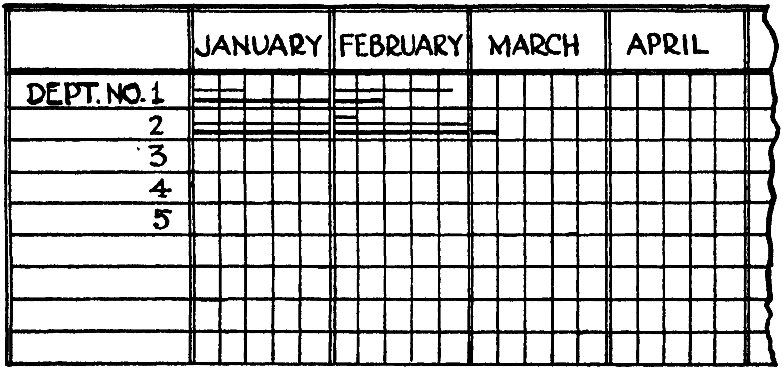


FIG. 131. GANTT CHART

The space is customarily divided into 5 parts, each part representing 20 per cent of the quota. If 90 per cent of the quota is accomplished during the next month, a bar of 4½ spaces in length will be drawn in the space allotted to February (see Fig. 131). A heavier bar immediately beneath the other bar is used to record the cumulative performance. Thus, a combined total of 7 spaces has been covered by the performance for these 2 months (2½ spaces for the 50 per cent performance of the first month, and 4½ spaces for the 90 per cent performance of the second month). A heavy bar is drawn across 7 of the spaces (see Fig. 131). It can be seen that this bar falls short of covering the space allotted to the two periods, and therefore the work accomplished was less than quota.

The record of Department No. 2 for the same 2 months shows that during the first month it accomplished 100 per cent of its quota, and during the second month 120 per cent of its quota. When the accomplishment exceeds 100 per cent of expected performance the first light line is drawn to the 100 per cent quota, and a second light line is drawn until enough spaces are covered to record the performance (in this case 6 spaces for 120 per cent). The second line is always drawn directly above the first. The heavy cumulative bar shows at a glance that the quota for the 2 months was exceeded.

Construction of the Gantt Chart.

1. Draw an appropriate grid or form. In drawing the grid, one line must be allotted to each progress record to be kept. Thus, if the chart is to be a machine record chart, there will be one horizontal line for each machine. An additional line may be allotted for the total for the entire department or factory.

   In preparing a performance record for the year 1935 for factory production according to type of product (such as each model of automobile), a line is allotted for each product and one for the entire production (at the top). Each horizontal line is divided into 12 equal parts, one for each month. These divisions may be set off by a heavy vertical line or a double line. Each of these spaces (months) will then be subdivided into 5 equal parts, each representing 20 per cent of the quota for the month. The 5 spaces within the month will then represent 100 per cent of the quota for each month.
2. Quotas are then established for each month of the year and for each model of automobile. If it is desired, these quotas may be entered on the Gantt Chart by putting the numbers on the chart.
3. After the results for the first month have been obtained

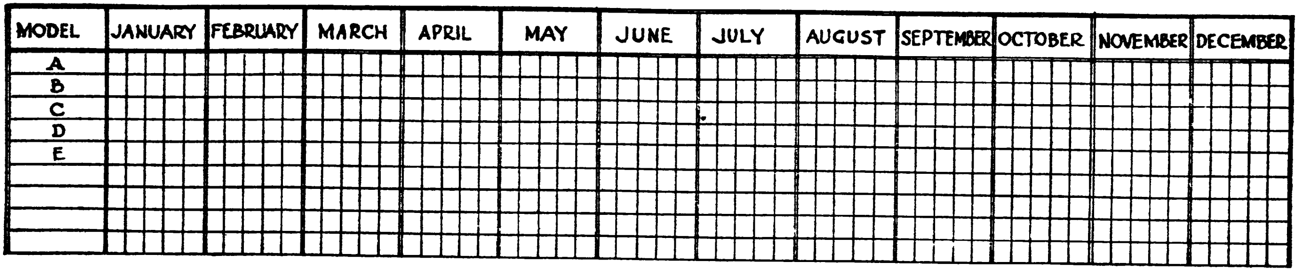


Fig. 132. Grid for Gantt Chart

they can be entered on the Gantt chart. If the results are as shown below, they may be entered as follows:

PRODUCTION OF CHEVEFORD AUTOMOBILES, JANUARY 1934

Model	Quota	Production	Per Cent of Quota
A	10,000	8,000	80
B	15,000	15,000	100
C	10,000	7,000	70
D	10,000	11,000	110
E	5,000	4,000	80
Total	50,000	45,000	90

Only 80 per cent of the quota was produced in Model A automobiles. A light line is drawn across 4 of the spaces for the month of January to record the performance for that month. A heavy bar is drawn immediately beneath

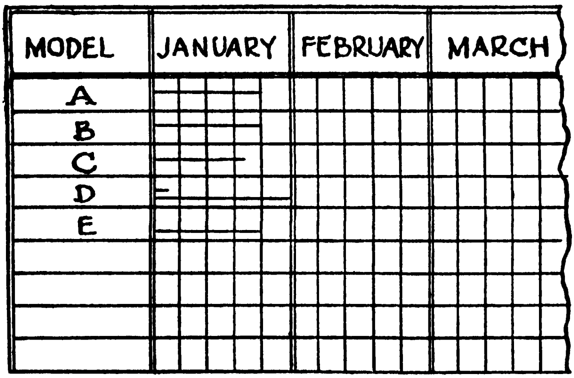


FIG. 133. AUTOMOBILE PRODUCTION, JANUARY 1934. (GANTT CHART)

it to indicate the cumulative performance and, since this is the first month, the number of spaces covered will be the same. In similar manner bars are drawn for the other models.

For the second month the results were as follows:

PRODUCTION OF CHEVEFORD AUTOMOBILES, FEBRUARY 1934

Model	Quota	Production	Per Cent of Quota
A	10,000	9,000	90
B	15,000	18,000	120
C	10,000	5,000	50
D	10,000	6,000	60
E	5,000	3,000	60
Total	50,000	41,000	82

The production of model A automobiles for the second month was 90 per cent of the quota for the month. A light line is drawn across the space for this month covering 4½ spaces (one space for each 20 per cent of the quota). The cumulative bar is prepared by cumulating the number of spaces covered by the light lines during the two months, January and February (in this

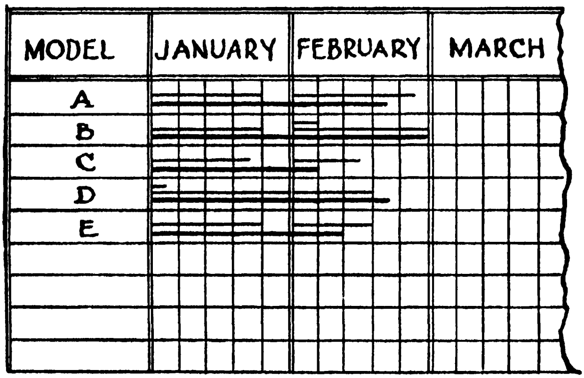


FIG. 134. AUTOMOBILE PRODUCTION, JANUARY–FEBRUARY, 1934. (GANTT CHART)

instance 4 and 4½). The heavy cumulative bar is then extended until it covers 8½ spaces. The production of model B for this second month was 120 per cent of the quota. The light line is drawn over the entire time interval (February) to represent 100 per cent of quota, and an additional light line is drawn over one

space to indicate the additional 20 per cent. The heavy bar is cumulated for this model as well, and is extended over 11 spaces (5 for January and 6 for February). A glance at the chart shows that the production of model A for these 2 months falls short of the quota, while that for model B is more than the quota for the period.

Symbols. A variety of symbols is used on the Gantt chart to record different facts about the progress of work and to indicate the reasons for the halting of a line at a particular point. Some of the symbols which are suggested for this purpose[2] are shown below:

V Chart entered to this point
Machine Record Chart
E Waiting for set-up
H Lack of help
M Lack of materials
O Lack of orders,
etc.
Work Layout Chart
┌ Date job is scheduled to start
┐ Date job is scheduled to be completed
Time required to make up for past delays

Legend. A legend to identify the symbols used in the Gantt chart is placed below the chart if necessary. However, if by constant usage those who read the chart become familiar with the symbols, it may be unnecessary to accompany the chart with a legend, since its information may be common knowledge to all who might use the chart.

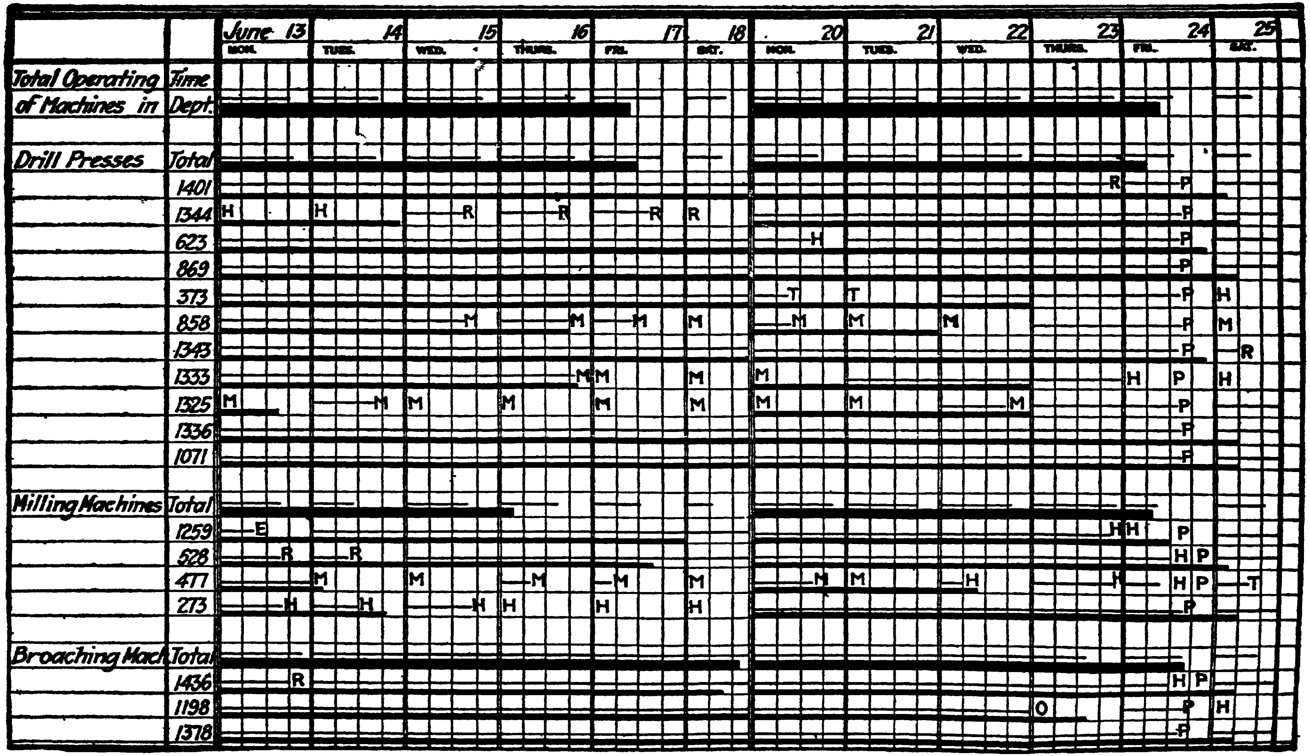
Machine Record Chart. The purpose of the machine record chart is to analyze machine idleness and indicate the reason for that idleness. The time interval, in this instance the day, is divided up

[2] From Clark, W., *The Gantt Chart*, New York, Ronald Press, 1922.

in units of time rather than per cent of quota. Thus, for an 8-hour day, the space is divided into 4 parts, each indicating a 2-hour period. The ratio of the line to the space will then indicate the proportion of the time the machine was in operation. If this space is blank, the machine did not run at all. Reasons for idleness may be indicated by means of symbols of the type outlined above. The heavy line in this type of chart indicates the cumulative running time for the week. A machine record chart is reproduced in Fig. 135.

Other Gantt Charts. There are numerous variations in the ways performance may be portrayed by means of these charts. These variations are dependent on the type of information to be recorded and the decision of the constructor of the chart as to the best means of simply, accurately, and clearly presenting the facts. Wallace Clark, on the *Gantt Chart*, presents various types of charts, including the man record chart, the load chart, the shipping chart, and the layout chart. These charts are similar to the progress chart used in the illustration on page 187, varying only slightly in construction with a change in symbols, numbers, and other identifications for those peculiar to the respective charts.

Advantages of the Gantt Chart. Although the Gantt chart was used originally in business for planning and production, and is still so used, it can be employed in any field of endeavor to depict performance in relation to time. The chart is simply constructed, it is easy to read and interpret, and, unlike other forms of graphs, it may be used to explain directly on the graph the causes for the failure of actual accomplishment to meet expected performance. Thus the cause may be identified and remedied. The simplicity of the Gantt chart is further seen in its movement and coloring. The chart always moves from left to right, the lines never cross, and only rarely are any colors used, since only black is necessary to depict the facts. Thus, the Gantt chart can be re-



From "Scientific Management in American Industry," The Taylor Society, Harper and Brothers, 1929

FIG. 135. MACHINE RECORD CHART

produced easily, clearly, and inexpensively. Another major advantage of the Gantt chart is that a great amount of information can be recorded on a single chart and yet permit of simple interpretation.

The Break-even Point Chart. The Break-even Point chart is a special type of graph of relationship used in management control. It presents the relationship between the per cent of capacity of operation (or volume of production) and costs.

In preparing a graph of this type an arithmetic grid is constructed. On the X (horizontal) axis a scale is arranged for per cent of capacity (or volume of production). On the vertical (Y) axis a scale is arranged for the volume of sales (or costs) corresponding to the per cent of capacity of operation. The scale on the X axis (per cent of capacity) will range from 0 to 100 per cent, while that on the Y axis will range from 0, the sales volume at 0 production, to the maximum sales value at 100 per cent capacity production.

The costs of operation at a given per cent of capacity are then grouped under two general headings, fixed costs, and variable costs. The fixed costs are those which do not change to any marked degree with a change in the volume of production, and include such costs as interest on the funded debt, depreciation, rent, insurance, maintenance, taxes, and similar non-variable costs. All costs which vary with the volume of production are grouped together.

The fixed costs may then be plotted on the graph, using the horizontal scale for the per cent of capacity production (or corresponding volume of production) and the vertical scale for the amount of the costs in dollars. Since these costs are relatively fixed, the plotted line will be horizontal and parallel to the X axis (see line A, Fig. 136).

By estimation, or by past experience, the variable costs are then determined for several levels of operation (per cent of capacity

of operation). These values are added to the fixed costs, thus obtaining the total cost of operation at various levels of operation. These points are located on the graph by reference to the two scales. A line or smooth curve may now be drawn through the plotted points. The curve will ascend across the face of the graph, since the total costs will rise with an increase in the volume of operation (see line B, Fig. 136).

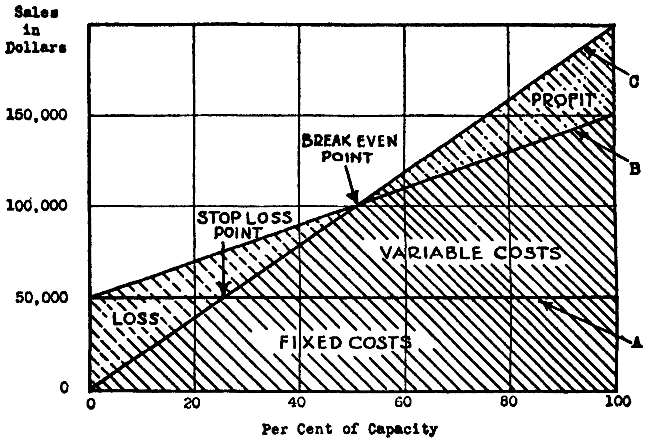


FIG. 136. BREAK EVEN POINT CHART

The cumulative cost line may assume the form of a straight line or the shape of a curve. If it is a straight line, its position may be determined by computing the total costs at only three levels of operation. If the total cost curve is not straight, its form can be determined only by securing a number of total cost figures and drawing a smooth curve through them.

The graph is now in the form of a band chart (see page 57) representing the costs of operation at various levels of production. The lower band represents the fixed costs, the width of the upper band represents variable costs, while the top line represents the total cost of production. The area under the cost curve may now be cross-hatched to represent the total cost. If desired, dif-

ferent cross-hatchings may be used to indicate fixed and variable costs. (For the construction of cross-hatchings see page 142.)

Sales, or income, at the various levels of production, are indicated by plotting a line of the same grid (line C, Fig. 136). This line will start at zero on both axes (since sales, or income, at zero per cent of capacity are zero) and ascend to the point corresponding to 100 per cent of capacity and the maximum sales or income. Since these two points are located at the lower left-hand corner of the graph and the upper right-hand corner, respectively, this line will form a diagonal across the grid. The sales line is below the cost line for part of the distance across the grid, and therefore at these levels operation will be at a loss. The area representing sales loss may be indicated by continuing the cross-hatchings of the cost area through this section in red.

If necessary, the horizontal scale may be in volume of operation rather than per cent of operation if capacity in the given type of business is indeterminable.

The point at which the income line crosses the cost line is known as the "break-even" point, since at this level of production income will just equal costs. After this point, the sales line rises above the cost line and the area between these two lines represents a profit. The profit area may be cross-hatched appropriately.

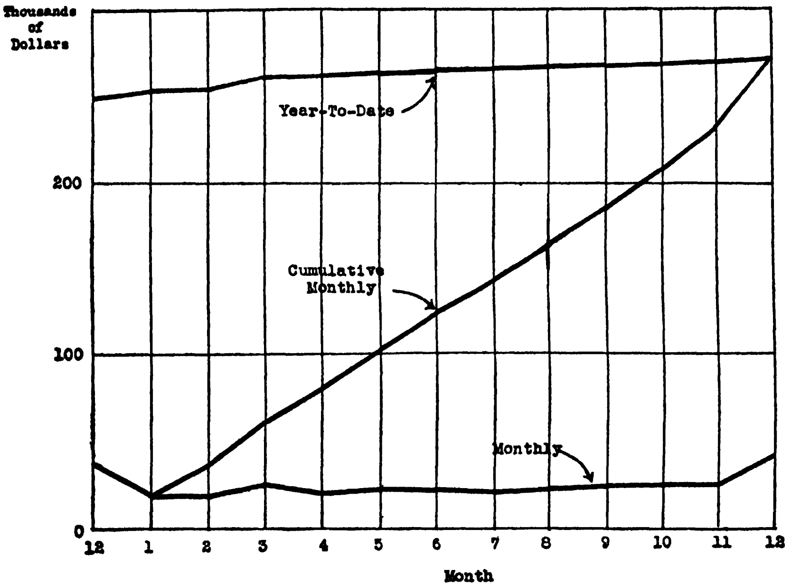
The point *a*, at which the income line crosses the fixed costs line, is known as the "stop loss" or "shut down" point, since the income from operations at levels of production below this point does not even cover overhead costs.

The break-even point chart may be applied to many other types of business concerns. Its use may also be adapted to factories, hotels, ships, restaurants, railroads, retail organizations, etc.

The Zee Chart. The Zee chart is a special type of line graph which acquires its name from the fact that the curves form a crude Z on the face of the chart. This chart is of great help to the business executive, since it presents a complete summary of

the data plotted. It generally deals with sales, production, or other similar data over a period of time.

The chart consists of three curve lines. The first represents the monthly (or weekly) sales, the second the cumulated monthly (or weekly) sales for the year, and the last the year-to-date sales



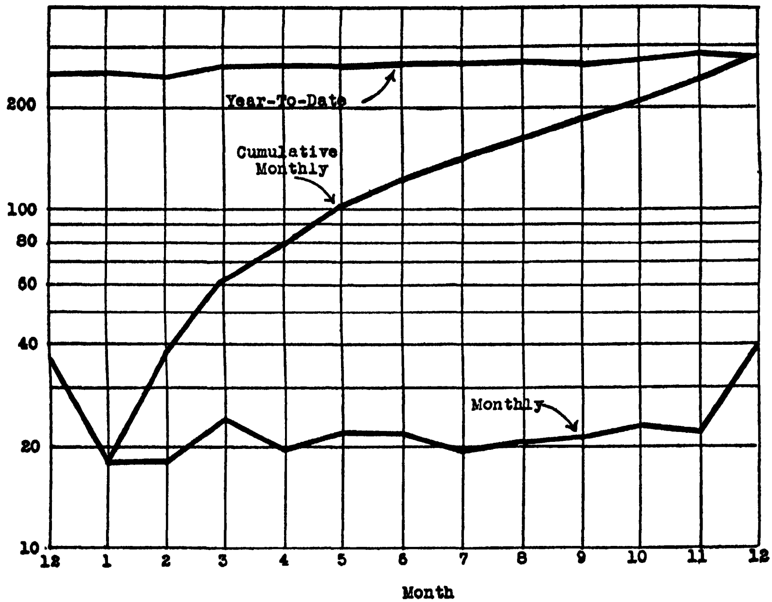
a.) Arithmetic Grid

FIG. 137. SALES OF F. W. WOOLWORTH CO., 1934 (ZEE CHART)

for the last 12 months (or last 52 weeks) including the present one.

The scale on the Y axis of the graph must be prepared so that the year-to-date sales be included as well as the monthly sales. If an ordinary scale is used, the variations exhibited by the monthly (or weekly) curve will be reduced until they are negligible. A much more satisfactory method is to plot the curves on semi-logarithmic paper (see Chapter V). When plotted on this type of ruling, the curves can be compared, since equal percentage changes will be given equal distances on the graph.

Comparisons of the trend from year to year can be made by placing the Zee charts side by side. For this reason, the last month of the previous year is generally used as the beginning period of the chart. However, the cumulative curve begins at zero for this



b.) Semi-Logarithmic Grid

FIG. 137. CONTINUED

point. When placed side by side, the monthly and year-to-date curves will then be continuous. A comparison of the seasonal variation can be secured by placing the Zee charts one above the other.

CHAPTER XIV

THE REPRODUCTION OF GRAPHS

Tracings—Hectograph Process—Mimeograph Process—Multilith Process—Blue Printing—The Photostat—Printing Processes—The Line Cut—The Half Tone: Rotogravure—Offset Printing—Reproduction of Color.

With the widespread use of the graph in recent years there has been a corresponding increase in the demand for copies of graphs. The amount of work required in the preparation of the original often makes reproduction by redrawing too arduous and expensive. With the modern methods of reproduction now available, it is a comparatively simple matter to reproduce readily and cheaply almost any graph.

The choice of which of the numerous methods to use is dependent on the number of copies desired, the type of graph to be reproduced, the speed required, the expense involved, and the purpose for which the reproduction is made. The more common methods which can be used to reproduce graphs are:

1. Tracing paper and tracing cloth.
2. Hectograph.
3. Mimeograph.
4. Multilith.
5. Blue Print.
6. Photostat.
7. Printing.

Tracings. Graphs may be copied directly from an original by means of tracing-paper or tracing-cloth.

Tracing-paper is a light weight, translucent paper which is placed over the original copy, and a tracing of the drawing is made on this sheet. The tracing may be prepared either in pencil or directly in ink. Since tracing-paper is thin, it crumples easily and therefore does not permit of frequent or hard erasure. Printed graph paper prepared on tracing-paper can be obtained in all of the standard types.[1]

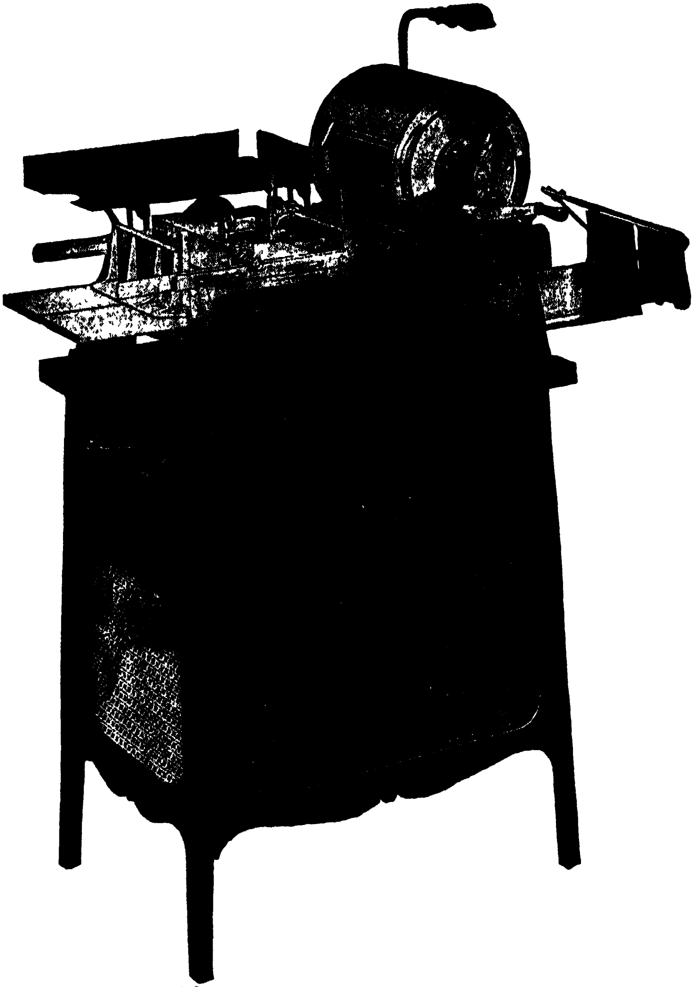
Tracing cloth is a starched translucent cloth which is used in a similar manner to tracing-paper. It is light blue in color, and since it is made of cloth it is durable. A finished tracing when prepared on tracing-cloth not only has a better appearance, but is more permanent than when prepared on tracing-paper.

Reproduction by tracing eliminates a considerable amount of the work in planning and measuring the original graph. Nevertheless, this method requires much time and effort, and is not generally used when more than one or two copies are required. Reproduction by tracing is the least expensive method for obtaining a small number of reproductions, if the time consumed in preparation is discounted.

Hectograph Process. There are a number of reproductive methods that make use of office equipment. These methods, including hectographing, mimeographing, and multilithing, require a specially prepared original. In the hectograph process, the graph is prepared with the aid of a hectograph pencil or ink on an ordinary sheet of paper. Typewritten information may be recorded by means of a typewriter with a hectograph typewriter ribbon. The drawing is then transferred to a gelatine pad by pressing the original against the surface of the pad. Copies may be prepared by placing sheets of paper on the pad. A copy of the drawing will then be transferred to these sheets.

Hectograph inks, pencils, and typewriter ribbons can be obtained in a variety of colors. The process can be used effectively only when a limited number of copies is desired, since, with use,

[1] Standard types of ruled graph paper are illustrated on pages 45, 46.



Courtesy of the A. B. Dick Co.

FIG. 138. THE MIMEOGRAPH MACHINE

the original impression fades rapidly from the gelatine pad. Generally, an original cannot be used for the preparation of more than forty or fifty copies.

Mimeograph Process. In the mimeograph process of reproduction, the original is drawn or fixed on a stencil by means of a stylus, while the lettering may be inserted on a typewriter with

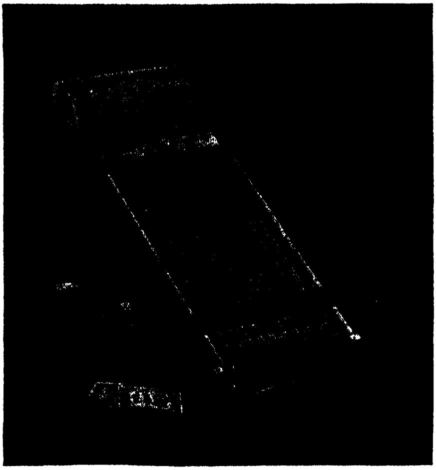
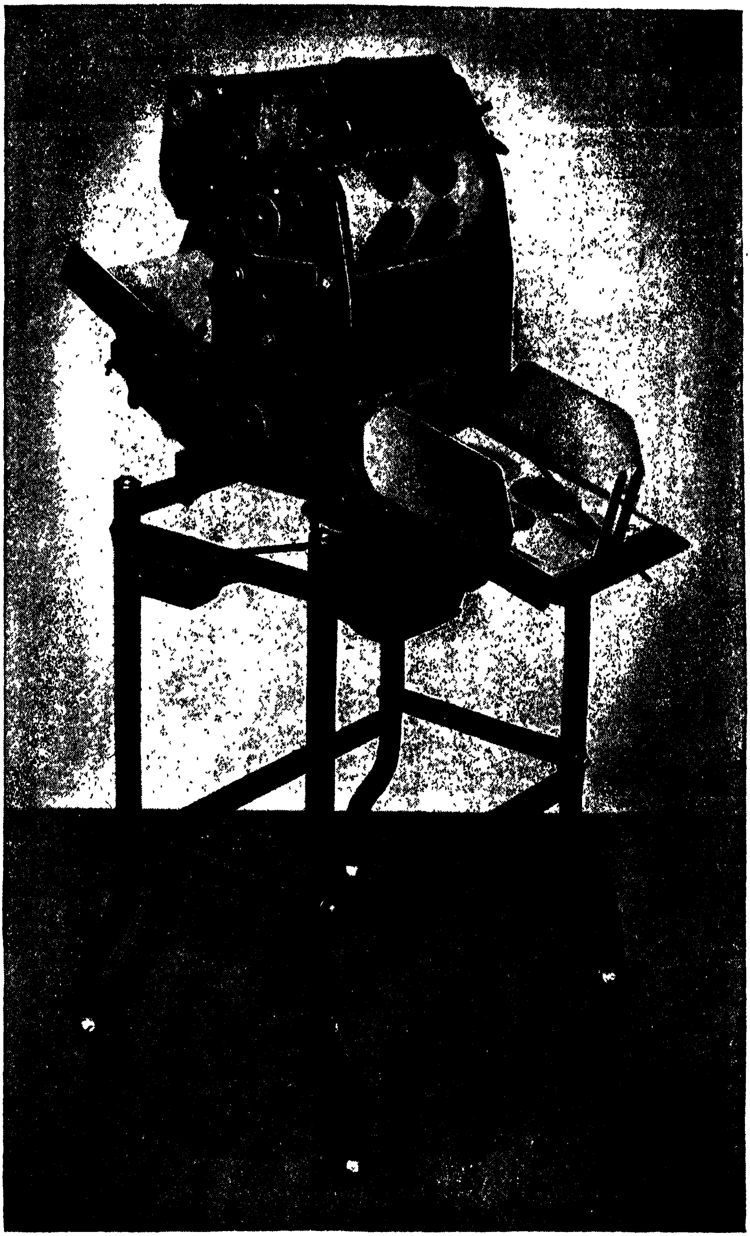


FIG. 139. THE MIMEOSCOPE

the ribbon removed. The stencil is placed in a rotary mimeograph machine over an inking attachment, and paper is fed through. An impression will be left on each of the sheets.

While the stencil can be used for the preparation of a large number of copies (generally not more than 4,000 clear copies), only certain types of paper can be used. The resulting reproductions are frequently not so clear as those produced by some of the other methods. The tracing of a graph on a stencil may be done with the aid of a mimeoscope. This is an opaque glass under which a strong electric bulb is placed. The graph is placed on top of the glass with the stencil above it. The light from the bulb passes through the graph paper and facilitates the tracing.

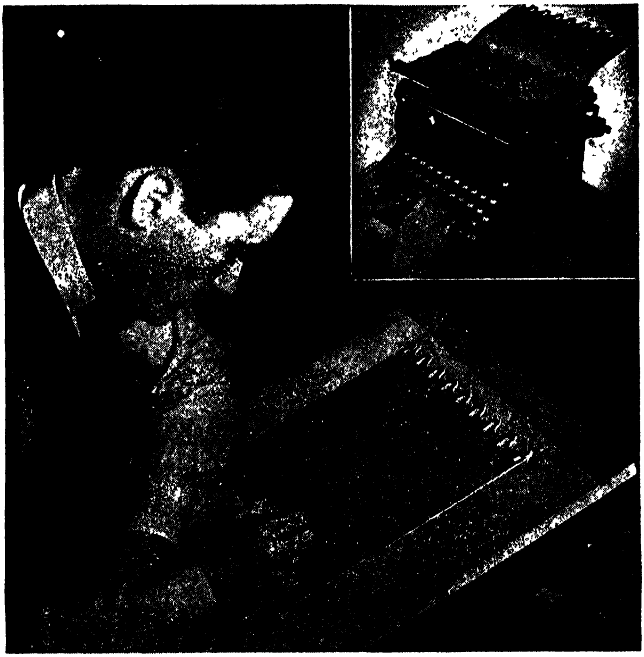


Courtesy of the Addressograph-Multigraph Corp.

FIG. 140. THE MULTILITH MACHINE

Multilith Process. The multilith process is a fairly recent development in the field of reproductive methods. The machine used for this method is larger and more complicated than the mimeograph machine (see Fig. 140).

The original drawing is made on a flexible zinc plate (see Fig. 141) by pencil, crayon, brush, typewriter, or other method. If



Courtesy of the Addressograph-Multigraph Corp.

FIG. 141. PREPARING THE MULTILITH PLATE

the drawing to be reproduced has already been prepared, it is transferred to the plate by a photographic process. The photostat camera, using a sensitized sheet of paper for film, has been used very successfully in conjunction with the multilith. The photostat camera must be equipped with a ground-glass background. The developed paper film is "burned" onto the multilith plate by arc light.

This method of reproduction is particularly desirable where a photostat camera is already owned, because it combines the economy of the photostat process with the ability of the multilith to produce a larger number of fine copies. After the drawing has been placed on the plate, the plate is wiped with a special solution. Erasures can be made with a red pencil eraser. Both line drawings and halftones can be reproduced by this method.

The process of reproduction is essentially similar to offset printing (see page 221). The image is placed on the plate with a greasy substance, and as a result the greasy lithograph ink will adhere to it. The rest of the plate has been moistened with water and therefore will repel the greasy ink. The image in ink is then transferred to a rubber blanket, and from this blanket to the paper.

Since this process is virtually a printing process, it can be used for color reproduction by preparing a separate plate for each of the primary colors needed. (See page 221 for a discussion of the technique of color reproduction.)

The offset printing process yields reproductions with a high degree of fidelity, and therefore the multilith process, which is a type of offset printing, produces very fine copies.

The cost of the process is extremely low, since the plates and paper films are inexpensive, and clear reproductions may be obtained from a single plate for as many as 25,000 copies.

Blue Printing. Although the blue-print process of reproduction is widely used by architects and engineers to prepare copies of building or machinery plans, it has not been widely used in other fields in spite of its value in reproducing diagrams. The original for the blue-print process is prepared on tracing-paper or tracing-cloth, and is then exposed to the sun or to an electric light over a piece of specially sensitized paper.

The exposed sheet of paper is then dipped in water and dried. Those areas which were shaded on the original will be white on

the blueprint, while the other areas will be blue. The reproduced diagram will then be of the nature of a "negative," white lines on a blue background. If a positive is desired, special papers are available which will produce copies with blue or black lines on a white background. A similar result may be accomplished by taking a blueprint of a blueprint. Unlike the photostat, blueprints can be obtained in very large sizes.

The Photostat. The photostat process is used to obtain excellent reproductions of a graph when only one copy or a few copies are needed. In this process the original diagram is photographed directly on a sensitized sheet of paper. The paper is passed through a developing process. The resulting reproduction is a negative (white lines on a black background). To obtain a positive, a photostat of the negative is made.

The photostat process is an invaluable aid in preparing graphs. Reductions and enlargements in any degree may be obtained by this process. The method may be used to prepare solid diagrams where the figures must be in a given proportion. The figures may be enlarged or reduced to the desired proportions quickly and inexpensively.

Another advantage of the use of the photostat is that, since a photograph is being taken, segments of charts may be covered (by placing slips of white paper over them), cut, or added to, and yet evidence of these additions or deletions does not appear in the finished picture. A photostat of an 8½x11-inch graph costs approximately 25 cents for the negative and 25 cents for the positive.

Printing Processes. All printing processes consist in making an impression in ink on a sheet of paper. Therefore they may be classified according to the type of method used to make the impression. The methods of making the desired ink impression,

classified according to the type of surface used to make the impression, are as follows:[2]

1. The relief process:
   This is the oldest and most common process. It makes use of a printing surface that stands out in relief against the background. The line cut and halftone engravings are examples of this process.
2. The intaglio process:
   In this process the diagram is etched below the surrounding surface. The rotogravure method is an example of the intaglio process.
3. The planographic process:
   Reproduction is made by means of diagrams reproduced in oil or grease on a surrounding moistened surface. Offset printing is a process of this type.

A more complete listing of the printing processes is as follows:[3]

1. The relief process:
   Hand-set type, linotype, monotype, intertype, woodcuts, line and halftone engravings, electrotype, nickeltypes, stereotypes.
2. The intaglio process:
   Etchings, steel and copperplate engravings, mezzotints, gravure, photogravure, rotogravure.
3. The planographic process:
   Lithography, artotype, collotype, gelatin, offset, heliotype, albertype, aquatone.

The Line Cut. The line cut and halftone, the most usual processes used in reproducing graphs, are made by a photoengraving process. In this process, a photograph of the original drawing is taken

[2] There are many processes which may be used to obtain this result. However, only those which are commercially important in reproducing graphs and diagrams are dealt with in this chapter.

[3] This classification is from Oswald, J. C., *How to Buy Printing Profitably*, New York; Employing Printers' Association.

on a sensitized zinc or copper plate, which is used in a camera in place of the more usual gelatine film or plate. The plate is then bathed in a prepared acid. Through a chemical reaction the exposure renders that part of the plate where lines appeared in the original drawing impervious to the action of the acid. Thus, the background is etched out by the acid bath, leaving the printing surface (the diagram reproduced in metal) raised above the surrounding metal of the background. The line cut which is produced by this method may be used to reproduce line drawings, but cannot reproduce shadings of the type that appear in photographs. Most of the graphs in this volume were reproduced by the line-cut method. If shadings are desired, they may be placed directly on the engraving by the engraver through the Ben Day process. The Ben Day shadings are produced mechanically.

The Halftones. Since the line cut cannot reproduce the shadings of the original drawing, it cannot be used to reproduce many types of illustrations, including photographs. A somewhat similar process may be employed for this purpose by use of the halftone cut. In this process, the original is photographed on to a sensitized metal plate. However, the photograph is made through a screen, which breaks up the picture into a myriad of tiny dots, some dark and some light. These dots are not seen readily in the printed picture, but can be discerned if the picture is examined with a magnifying glass. The halftone prints best on a hard-surfaced paper, while type prints best on a soft-surfaced paper. If both printed material and illustrations are to appear on the same sheet, this method does not give the best result. Books containing many illustrations are often printed on hard-surfaced paper for this reason.

Rotogravure. The rotogravure process involves the photographing of the original through a halftone screen and *etching the diagram* into the plate. This results in a diagram which is cut

into the plate below the surrounding metal. Ink is placed in these impressions in the printing-press. When the plate is pressed against the paper, some of the ink adheres to the paper, thereby producing the picture. A less expensive uncoated paper may be used for rotogravure printing, resulting in a considerable saving in paper costs. The halftone "dots" are less visible to the eye when produced by this process, thus yielding a better reproduction.

Offset Printing. The offset printing process is an indirect planographic method. In this method, the diagram is fixed on a porous surface by photographic methods. The design is then transferred to a rubber blanket. The surface surrounding the design is moistened with water between impressions. The greasy ink used adheres to the unmoistened surface and is repelled by the moisture on the rest of the surface. An impression can be made by pressing the rubber blanket against the surface of the paper. The resulting reproduction is produced with a high degree of fidelity. This is due to the fact that the rubber is pressed into the most minute crevices of the surface of the paper. The original plate can be used to print satisfactorily as many as 50,000 copies. The reproductions obtained by this method are comparatively inexpensive, due to the large number of copies that may be obtained from an original plate, and since an uncoated and cheap paper may be used.

Reproduction of Color. The only process by means of which an original can be faithfully reproduced in the original colors is color printing.[4]

If only one or two copies of a graph are desired, the original may be traced, photostated, or otherwise duplicated, and the copies colored by hand.

For the reproduction of a quantity of colored charts, resort

[4] Since the multilith process is a printing process, it may be used for color reproduction. (See page 216.)

must be had to the printing process. This process is much more expensive than ordinary reproductive printing. It costs from three to six times as much to print in colors.[5]

Color printing is accomplished by printing one color over another. Different colors are obtained by combining the three primary colors—blue, red, and yellow.

The color-reproduction process consists of the superimposition of printings of these colors in addition to black. This process is therefore called a four-color process.

A photograph of the original colored graph is taken through a color filter, which eliminates all colors except the one for that particular plate. The plate is then prepared on the basis of this photograph. The four plates are then printed on the same surface, using approximately colored inks.

The four-color process cannot be used for reproducing line drawings, although areas can be filled in with colors by means of specially prepared plates. Halftone plates are generally used in this process.

[5] The cost will vary with the number of primary colors used.

CHAPTER XV

STATISTICAL TABLES AND REPORTS

Types of Tables—General-purpose Tables—Special-purpose Tables—Construction of the Table—Title—Arrangement of Data—Column Captions—Units of Measurement—Symbols—The Stub—Footnotes and Prefatory Notes—Source—Totals—Rulings—Planning the Table—The Report—The Elements of the Report—The Report Cover—Letter of Transmittal—Body of the Report.

THE graph is generally accompanied by a statistical table containing the data from which it was constructed, since it is not intended that the individual values be read off a graph with any degree of accuracy. The statistical table is a systematic arrangement in convenient form of numerical data which are presented in contiguous columns and rows for comparison, analysis, or reference. A table should be entirely self-sufficient and no supplementary textual explanation should be necessary for understanding it.

Types of Tables. Statistical tables, although of varied types and complexity, have been classified according to purpose, as general-purpose (primary or reference) tables and special-purpose (derived or summary) tables.

General-purpose Tables. The general-purpose table is constructed to provide ready reference and also to serve as a source of original reference from which other (special-purpose) tables may be constructed. As a rule, the figures in the general-purpose table are not

rounded off, but are recorded in their full value. The United States Census volumes constitute a good example of a general-purpose table.

Special-purpose Tables. The special-purpose table, as its name implies, is designed primarily to present numerical data in such a way as to emphasize specific relationships or to bring out certain characteristics of the data. It is this type of table that generally accompanies the graph. Since the special purpose table permits the presentation of selected material in simple form, it is permissible to round off numbers. The presentation of the selected materials in a special purpose table in a small space facilitates the interpretation of the data. The data must be carefully arranged so that the desired comparisons can be seen easily.

Construction of the Table. Statistical tables should be so constructed as to present data clearly and simply in a systematic order.[1] A knowledge of the parts of a table will facilitate such construction. The components of a typical table and the terms assigned to them may be seen on p. 214.

The following characteristics of parts of the model table should be noted.

1. The completeness of the title, explaining in order what, where and when.
2. The base period for the index number (1913 = 100) is placed immediately beneath the title.
3. The column captions are worded briefly, thus facilitating the reading of the table and contracting its width.
4. The footnote in this table indicates additional information pertaining to one column. The footnote is located at the left bottom of the table, below the body of the table and above the source.

[1] The exceptions to the generally accepted procedure of table construction may be justified by the particular purpose for which a table is being constructed.

Title →

INDEX OF RETAIL FOOD PRICES AND
THE COST OF LIVING IN THE UNITED STATES
1919–1933

1913 = 100

Boxhead → ← Column Captions

Year	Index of Retail Prices	Index of Cost of Living*
1919	186	199
1920	203	200
1921	153	174
1922	142	170
1923	146	173
1924	146	173
1925	157	178
1926	161	176
1927	165	172
1928	154	171
1929	157	171
1930	147	161
1931	121	146
1932	102	132
1933	100	135

Stub →

Footnote → * As of December.

Source → Source: United States Bureau of Labor Statistics; *Statistical Abstract of the United States*, 1934; pp. 289–292.

5. The source in the model table includes the compiling agency, the publication in which the data appeared, the date of the publication, and the page reference, in that order.
6. The data are arranged from the earliest year to the latest.
7. The model table is ruled in the following manner:

   A single border line separates the body of the table from the title above and the footnote and source below.[2]

   The column caption as well as the stub are separated from the rest of the table by double lines. However, a single heavy line may also be used for such a purpose.

[2] In typewritten or printed tables it is not necessary to enclose completely a table with a border.

The column captions are separated by a single line. If totals are included, they are also separated by a single line.

8. The years in the stub, and the corresponding figures in the other columns, are spaced so that the data in five year intervals may stand out. Spacing facilitates interpretation of tables.

Title. The title of a table should be self-explanatory and should indicate the subject covered, the area from which the data was drawn, and the period of time covered. The title should be as brief as possible without sacrificing completeness, clarity, or accuracy.[3]

Arrangement of Data. A careful arrangement of the items in a table is important, since it will facilitate the reading of the table, permit the emphasizing of particular groups of data, and facilitate analysis and comparison.

The arrangement of the items in a table runs from the top to the bottom for the stub, and the left to the right for the column captions. A number of arrangements are possible.

1. alphabetical
2. geographical
3. chronological
4. by magnitude
5. by customary classifications

The columns in the table are generally arranged so that the sets of data to be compared are placed next to one another. The table should always be arranged so that there are a minimum number of columns. In the alphabetical arrangement the items are arranged in the customary alphabetical order according to the names of the classes of data, while the geographic arrangement

[3] For a more detailed discussion of the principles of title construction see page 16.

lists the items in systematic order according to their customary classification, which is usually geographical. The alphabetical and geographical arrangements are used for reference tables only, since they do not tend to facilitate comparison.

The most usual arrangement in special-purpose tables is chronological order. In presenting data chronologically the earliest year is indicated at the left for the columns, and at the top for the new headings in the stub. However, where the most recent period is of unusual interest, as when the figures are published for the first time, the latest figure may be listed before any of the others. In this case, it may be separated from them by a double or heavy line.

An arrangement of items according to their magnitude or size is commonly used. In this instance, common practice dictates that the largest figure appears at the top of the column and the others be arranged in order of size. The row captions will then fall into the order created by the size of the values in the group. Where the row captions themselves are numerical, as in the case of the frequency distribution (in the form of class intervals), they may be arranged in order of size, in this case from the smallest at the top to the largest at the bottom, for the rows, or the smallest at the left to the largest at the right, for the columns.

There is a customary arrangement for many types of data which do not follow any serial arrangement. For instance, the classification men, women, and children, is rarely listed in the order, women, children, and men.

Column Captions. The heading of each column used to identify the data contained in it is known as the column caption. It should contain as few words as possible and explain exactly what the data in the column represent. If desired, the columns in the table may be numbered or lettered to facilitate text reference. A "miscellaneous" column should be placed at the end of the right side of the table.

Units of Measurement. Although it is a common practice when all of the data in the table are in the same units (such as thousands of gross tons) to indicate the unit immediately beneath the title, a better practice is to show it in the boxheads. If the units differ for the various columns, each column caption must include such information. The use of symbols or abbreviations (such as $, lb., etc.) should be avoided. Column captions should contain these notations, as "In dollars," "In pounds," etc.

The Stub. That part of the table containing the row captions is known as the stub. The preparation of the captions for the stub follow the same rules as for the boxhead. The stub (as well as the boxhead) is generally separated from the body of the table by means of a double line.

Items in the stub should be arranged in groups according to the type of data being used. The reading and interpretation of monthly data, for example, is aided when the months are grouped by quarterly or six-monthly periods.

Footnotes and Prefatory Notes. When an additional specification or indication of the inclusion or exclusion of items for all of the data in the table is necessary, the explanation is given in the form of a prefatory note which appears in small type just above the boxhead of the table.

When the explanation or amplification relates to one item only, or to a group of items, such as a column or row of figures, it is given in a footnote and is placed immediately beneath the table and just above the source. Footnote indications should be in the form of a symbol, #, *, or, as a second choice, a letter of the alphabet. A number should never be used, since it may be interpreted as a part of the table.

Source. When the data in the table is not primary—that is, when it is obtained from other sources—the source of the data should

be indicated. Denoting the source in this manner will lend authoritativeness to the figures and enable the reader to seek the original source if further information is desired. For this reason it is advisable, although not absolutely essential, to indicate specifically where the data in the table may be found. The source is placed immediately beneath the table to the left.

Totals. Since it is customary, when adding figures, to place the totals for columns at the bottom and the totals for rows to the right, the totals in tables appear in these same positions.

Rulings. In order that a table be used effectively, it should be ruled with lines to serve as a guide to the eye in examining figures. The columns should be separated by single lines. The stub and the boxhead should be separated from the figures by means of double or heavy lines, especially in non-printed tables. The boxhead is boxed in by a line drawn immediately above and below it. Totals are separated from the other items in a column by a single line, and a line should be drawn across the bottom of the table after the last figure or total. The arrangement of the ruling may be seen on page 214, as well as in the other tables in this volume. In typewritten tables it is not essential, although it is good practice, to separate the columns by means of vertical lines. All lettering on the table should be placed horizontally so that it can be read without difficulty.

Planning the Table. Much time will be saved if the arrangement of space for the table be apportioned before the table is drawn. This apportionment must include a consideration of:

1. The size of the paper and the desired margins both horizontal and vertical.[4]
2. The length of the longest word, phrase, or numbers in the

[4] The reduction and enlargement of tables by photostatic process, etc., is discussed in Chapter XIII.

stub, since this stub will influence the space apportioned to the stub.

3. The length (as typed, printed, or lettered) of the longest set of numerals in each column, since they may influence the space devoted to each column.
4. The length of the table as determined by the number of rows. In obtaining the total length of the table, the number of rows in the title, in the boxhead, the body, the footnotes, and in the source, should be considered.

The Report. The graph is seldom an entity in itself. It is generally used for the purpose of illustrating a statistical tabulation. In its most effective form it is used to visualize and simplify the complex data in a book, article, or report.

Since graphs commonly appear as part of a statistical report, the preparation of reports supplements the work of graphic presentation.

The effectiveness and success of the statistical report, of which graphs are a very important element, depend largely upon the preparation of the report. It is attention to the details of preparation that makes the report workman-like in appearance and greatly aids in delivering the message intended.

The Elements of the Report. The contents of the report will, of course, vary considerably with its elaborateness and formality, varying from a simple memorandum to a printed publication.

The contents, of an elaborate report are as follows:

A. Cover
B. Title Page
C. Letters of Authorization, Transmittal, and Approval
D. Table of Contents
E. Table of Charts and Illustrations
F. Preface and Foreword

G. Body of the Report
H. Appendix
  1. Statistical Tabulations
  2. Other supporting materials.

The Report Cover. The statistical report is generally bound with an appropriate cover, which will vary considerably in its form with the degree of formality and the type of report. An ordinary manila folder may be used as a cover, or any heavy tough paper folded to an appropriate size.

The type of paper used by lawyers to bind contracts (known as manuscript paper) is satisfactory for small reports and memoranda. This cover is made of light-blue paper, the top of which (about one inch) is folded over the top of the report. It is fastened into position with a clipping machine. Bindings made of real or imitation leather are obtainable for covering reports.

Letter of Transmittal. Business, governmental, and other similar reports are generally accompanied by a formal letter of transmittal addressed to the appropriate official. In the more formal reports, letters authorizing and accepting the report were also included.

Body of the Report. The body of the report generally begins with a summary or synopsis of its contents. This is followed by a detailed discussion of the materials of the report, and it generally ends with a discussion of conclusions and recommendations. Since graphs are illustrative of details contained within the text, they are included in the body of the report. The appendices include the more complete statistical tabulations, a description of the method used, and other supplementary details.

INDEX